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THESIS

ANALYSIS OF MODERN ANALOG AND DIGITAL COMMUNICATION CHANNELS FROM A MANAGER'S PERSPECTIVE

by

John A. Brouse, Jr.

March 1984

Thesis Advisor: Daniel C. Bukofzer

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SECURITY CLASSIFICATION OF THIS PAGE (*When Data Entered*)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS
BEFORE COMPLETING FORM

1. REPORT NUMBER

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (*and Subtitle*)
Analysis of Modern Analog and Digital Communication Channels from a Manager's Perspective

5. TYPE OF REPORT & PERIOD COVERED
Master's Thesis
March 1984

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(*s*)
John A. Brouse, Jr.

8. CONTRACT OR GRANT NUMBER(*s*)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Naval Postgraduate School
Monterey, CA 93943

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
Naval Postgraduate School
Monterey, CA 93943

12. REPORT DATE
March, 1984

13. NUMBER OF PAGES
183

14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)

15. SECURITY CLASS. (*of this report*)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)
Approved for public release, distribution unlimited

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)
Antennas
Waveguides
Transmission Lines
Communication Channels

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

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Analysis of Modern Analog and Digital Communication Channels from a Manager's Perspective

by

John A. Brouse, Jr.
Lieutenant Commander, United States Navy
B.S., University of Utah, 1974

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN TELECOMMUNICATIONS SYSTEMS MANAGEMENT

from the

NAVAL POSTGRADUATE SCHOOL
March 1984

Thesis
S18953
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This thesis presents the fundamental communication transmission principles which define the performance characteristics of the transmission channel. The subject matter is divided into three categories (1) antennas, (2) transmission lines, and (3) waveguides. The scope of presentation is results oriented rather than the traditional theorem and proof approach. While the results are quantitative in nature, consideration of the underlying principles as well as the advantages and disadvantages associated with each transmission channel are presented. Although technical factors are often the basis upon which decisions regarding communication systems are made, it is evident that the telecommunication systems manager must understand the fundamental principles of the transmission channel in order to effect viable solutions to telecommunications management problems.

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I. INTRODUCTION

The advent of the Information Age, coupled with an upheaval in the world economy over the past decade, have placed an ever-increasing emphasis on the corporate telecommunications systems and on the management of those systems. All telecommunications systems are designed to convey information from one point to another. In almost every case, the information content of the signal is not used by the telecommunications system itself. Rather, the system is designed to accept input data and structure it into a format that can be quickly, economically, and accurately transmitted to a destination. The information transmitted over telecommunications systems ranges from voice, telemetry, and facsimile data to complex interleaved data messages. Furthermore, telecommunications systems vary considerably in their designs and components with regard to technology, electronics, and methodologies. Despite the fact that there is such a wide range of input data and large diversity in the design of telecommunication systems, they have significant features in common. Fundamentally, all telecommunications systems exist to transmit information from one point to one or several points. Moreover, when viewed functionally, these systems have common structures.

Any telecommunications system consists of three basic functional components. The first component, the transmitter,accepts an input signal, modulates a carrier signal, and provides the power required to transmit over the communications channel. The second component, the channel, provides the path over which the signal travels. The last component, the receiver, extracts the signal from the channel, demodulates the carrier, and delivers the restructured original signal as its output. [Ref. 1].

While advancements continue to be made throughout the electronics industry, for example receiver and transmitter technology, there is less that can be done to increase the speed and frequency response of present day communications channels. The result is a greater need for an understanding of communication channels by telecommunication systems managers. The purpose of this thesis is to present the fundamental communications signal transmission principles in a manner that is relevant to the manager's systems approach to decision making.

A. APPROACH

No study of an intrinsically technical subject is approachable without mathematics. However, this thesis is written from a managerial perspective and, while essentially mathematical in nature, is limited in scope to emphasize areas considered fundamental to the underlying concepts of signal transmission in the field of communications. The approach therefore tends to be results oriented vice a traditional rigorous presentation of theorems and proofs. Additionally, presentation of each subject area begins with simple, ideal situations and progresses logically to the more complicated, general case.

B. AREAS OF STUDY

A large majority of the publications pertaining to communications signal transmission grossly characterize the electromagnetic propagation channels as either guided or unguided. Generally, in an unguided channel an electromagnetic field is generated by an antenna and propagates freely in a medium with no attempt to control its propagation pattern. A prime example of the unguided channel is the space channel in which the medium involved may be free

space, the atmosphere, or the ocean. In the guided channel, the communications signal is also transmitted as an electromagnetic field; however, its propagation is restricted to the confines of a closed path or "pipe" from the transmitter to receiver subsection of the communication system. Guided channels can be further subdivided into three categories which include 1) transmission lines, 2) waveguides, and 3) fiber optics. A less traditional and more subtle method of differentiating the propagation channels is by the number of conductors involved. There exist numerous cases whereby signals transmitted via antenna systems behave as guided waves [Ref. 2] thus, this thesis groups transmission channels according to the latter method as follows:

1. Antennas : zero conductors with energy propagating in the space between transmitting and receiving antennas.
2. Waveguides : one conductor with energy propagating within the boundaries of the guide.
3. Transmission lines : two or more conductors.

The study of the transmission channels formally begins with Chapter 2 which focuses on antennas. Noiseless systems are first discussed with emphasis being placed on the development of the power budget equation. Line-of-sight communication distances are explored and applied to a one-hop microwave system. Also covered in this section are satellite communication principles as they relate to the power budget equation. The concept of system noise is then introduced and an analysis of a multi-hop microwave relay system is presented.

Chapter 3 deals with transmission lines and is presented from a circuit theory viewpoint. Discussion begins with the low frequency model and development of the propagation

constant in terms of signal attenuation and phase shift or time delay is undertaken. The skin effect model is also explored. The chapter concludes with an introduction to a transmission line's digital transmission rate.

The waveguide is presented in Chapter 4 which begins with an introduction to the principles of electromagnetic field propagation. The plane wave is considered and the guide's modes of operation defined. A short section is devoted to the determination of guide cutoff frequency. Signal attenuation and time delay are developed through analogies to transmission line characteristics. A brief discussion of fiber optics, which is considered a special class of waveguides ends the chapter.

The concluding chapter of this thesis investigates the major advantages and disadvantages associated with the various communication transmission channels. Conclusions regarding the relative importance of communication channel considerations are presented in the context of a total telecommunications management scheme.

C. CRITICAL ASSUMPTIONS AND PRINCIPLES

Nearly all of communications theory is rooted in the science of mathematics. To underscore the fundamental concepts of signal transmission much of the rigorous mathematics encountered in a more traditional approach to the subject has been minimized. To this end several key assumptions and transform analysis principles are applied throughout the remaining chapters of this thesis.

1. Fourier Series

Communication transmission systems, and communication systems in general, deal primarily with time functions. These time functions or waveforms can be represented as a

summation of purely sine and cosine functions according to the Fourier Series expansion:

$$f(t) = a_0 + \Sigma[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t] \qquad \text{(eqn 1.1)}$$

where ω_0 is the fundamental frequency of the signal. Thus, the Fourier Series expansion is the sum of all frequencies which comprise a given signal. These signals are viewed as a combination of a fundamental frequency plus all harmonics of that frequency. The a_0 term represents the dc level of the signal. Since all frequencies present in the signal will not be in phase at all points in time, they are written as having two components, sine and cosine. Then, for any given harmonic n, the a_n and b_n terms represent the magnitudes of the sine and cosine portions of that harmonic. Under normal circumstances, discussions of communication systems tend to not deal with the time domain behavior of the signal but rather with the frequency domain behavior of the signal. Therefore one must be able to convert or transform time function signals to frequency domain signals. The tool so used is the Fourier Transform and is defined as

$$F(\omega) \triangleq \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \,. \qquad \text{(eqn 1.2)}$$

It can then be seen that every function of t has with it a corresponding function of frequency, ω. Likewise the Inverse Fourier Transform,

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d \qquad \text{(eqn 1.3)}$$

shows that for every function of ω there exists a corresponding function of t. [Ref. 3].

Communication systems are intended as a vehicle for the transmission of information whose parametric values at the systems level are relative to the frequency domain. The Fourier Transform is the single most important tool which allows movement from the time domain to the frequency domain and is used extensively to describe the frequency content of a given signal. As a result, a system analysis can be conducted which describes a system's effect on the information signal in terms of signal frequencies.

2. Linear Systems

In general, a system is defined as a set of rules that associates every input time function with an output time function as shown in Figure 1.1. The source signal is designated f(t) with g(t) representing the output signal or system response due to the input. This is usually written as $f(t) \longrightarrow g(t)$.

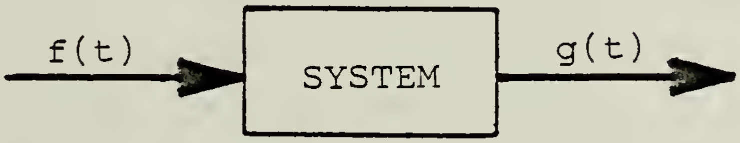


Figure 1.1 Block Diagram of a System.

This thesis is based on first order analysis and therefore all systems discussed are assumed to be approximately linear in nature. To this end, the definitions which follow apply.

A system obeys superposition if the output or response of the system due to a summation of inputs is equal to the sum of the responses due to the individual inputs.

That is, given that $f_1(t)$ results in a response of $g_1(t)$, and that $f_2(t)$ causes an output of $g_2(t)$, then the output due to $f_1(t)+f_2(t)$ is $g_1(t)+g_2(t)$. In mathematical notation, superposition is defined as [Ref. 4]

$$f_1(t)+f_2(t) \longrightarrow g_1(t)+g_2(t).$$

Similarly, a system is considered linear if it obeys superposition. Combining the concepts of superposition and linearity, it can be shown that for all values of the constants a and b [Ref. 5]

$$af_1(t)+bf_2(t) \longrightarrow ag_1(t)+bg_2(t).$$

All systems discussed are considered time invariant. That is to say that the system response due to an input is independent of the actual time the input occurs. In other words, a time shift in the input signal results in an equal time shift in the response. Thus it can be shown that if $f(t) \longrightarrow g(t)$, then $f(t-t_0) \longrightarrow g(t-t_0)$. Systems composed of purely resistive, capacitive, and inductive elements are time invariant provided their component element values do not vary with time.

3. System Function

An important and helpful concept in systems analysis is the impulse or Dirac delta (δ) function. Simply stated the impulse function is defined as

$$\delta(t) = \left\{ \begin{array}{ll} \infty & \text{for } t = 0 \\ \\ 0 & \text{for all other } t \end{array} \right\}$$

such that the total area under the impulse function is unity. Mathematically then

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

which also, as it turns out, is the Fourier Transform of the impulse function. It can be seen that any signal in the time domain can be divided into equal time segments. Furthermore, these time segments can be viewed as becoming infinitely small such that the segment approaches a point in time. Therefore, any time signal can be considered to be a series of impulses. The purpose of systems analysis is to determine the response the system has to the input signal. If the input signal is viewed as a series of impulses and the system is considered linear, then the response due to an impulse is all that is needed to describe the system. However, as previously stated, the over-riding factor in an analysis of a communication system is the system's effect on input frequencies. This is accomplished by studying the system's response to an impulse. [Ref. 6].

If the impulse response of a system is called h(t), then the Fourier Transform $H(\omega)$ indicates the system effect on the impulse and is called the system function or the transfer function. Simply stated the transfer function, $H(\omega)$, of a system describes the effect the system has on an input, $F(\omega)$, to produce an output, $G(\omega)$, and is the ratio of the output to input in terms of frequency. Mathematically,

$$G(\omega) = H(\omega)F(\omega).$$

4. Decibel and Logarithmic Notation

As will be shown throughout the thesis, one of the more important aspects of the communications channel analysis is the subject of signal power and how the system affects it. Generally, the system either amplifies or attenuates the signal power so that a system's or component's gain (loss) can be written as the ratio of output power to input power. Letting G denote the gain and P_{out} and P_{in} represent output and input power respectively, then

$$G = \frac{P_{out}}{P_{in}} .$$

Thus if the power going into an amplifier is 1 watt and the output is 100 watts the power gain is 100. Normally, however, communications system characteristics are expressed in terms of decibels (dB) vice the raw gain or attenuation figure. The dB notation is used extensively throughout the thesis and so requires some explanation.

The decibel is defined as one-tenth of the fundamental division of a logarithmic scale for expressing the ratio of two amounts of power [Ref. 7]. But, in general, any unitless quantity in the context of system analysis can be expressed in decibels according to the following equation where N denotes the number of dB's and x the unitless quantity,

$$N = 10\log x.$$

Normally then, system analysis compares some initial power level to the final system/component output level. This

initial reference point can be given in terms of either watts or a dB reference level with 1watt equal to 0dBW or 1milliwatt equating to 0dBm.

Additionally, because dB notation is used so often, the laws of logarithms are an integral part of analysis development. Logarithms derive their usefulness in computation since they allow multiplication, subtraction, and exponentiation to be replaced by simpler operations of addition, subtraction, and multiplication respectively according to the following [Ref. 8]:

$$\log(xy) = \log(x) + \log(y)$$

$$\log(x/y) = \log(x) - \log(y)$$

$$\log(x)^{y} = y\log(x)$$

II. ANTENNAS

In communication systems where the transmitter and receiver are not physically connected by conducting elements, the carrier waveform is propagated from the transmitter to the receiver by use of antennas. An antenna is simply a transducer which converts electronic signals into electromagnetic fields and vice versa. Thus a signal begins as an electronic signal at the transmitter and is converted to a propagating electromagnetic field at the transmitting antenna. This electromagnetic field propagates as an ever expanding plane wave, through the medium separating the transmitting and receiving antennas. As a result, the amount of power contained within a given area of the transmitting medium (field density) decreases as the distance between the transmitting antenna and the area under consideration increases. Thus as the distance between the receiving and transmitting antennas increases, the power of the electromagnetic field impinging upon the receiving antenna decreases. The receiving antenna, in turn, converts the electromagnetic wave back to an attenuated (reduced) version of the electronic signal produced in the transmitter. It should be noted that in addition to the intended transmission signal, other signals including background noise are present and impinge on the receiving antenna inducing unwanted voltages. Therefore, not only is the communication signal being received attenuated, but also it is inherently noisy as depicted in Figure 2.1 .

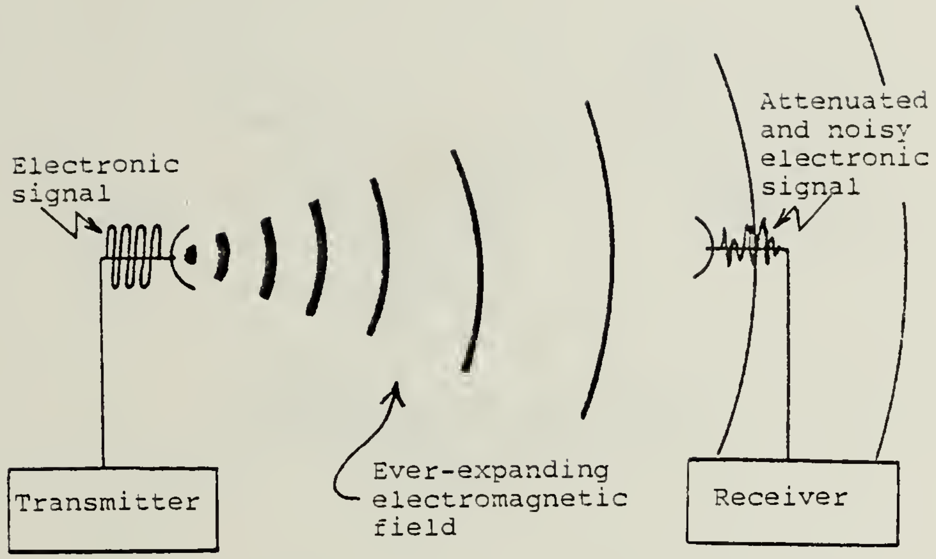

Electronic signal
Attenuated and noisy electronic signal
Ever-expanding electromagnetic field
Transmitter
Receiver

Figure 2.1 Signal Transmission via Antennas.

A. TRANSMITTING ANTENNAS

A transmitting antenna converts an amplified carrier signal into a propagating electromagnetic field. This field is transmitted by the antenna as a propagating plane wave with a prescribed polarization and spatial distribution of its field power density. The spatial power distribution of a transmitting antenna is described by its antenna pattern which is defined as

$$W(\phi_\ell,\phi_z) = \begin{array}{l}\text{power transmitted per unit solid}\\ \text{angle in the direction } (\phi_\ell,\phi_z)\end{array} \qquad \text{(eqn 2.1)}$$

where (ϕ_ℓ,ϕ_z) are the elevation and azimuth angles relative to a fixed coordinate system whose origin is the center of the antenna (see Figure 2.2). The antenna pattern indicates the amount of field power that will pass through a unit solid angle in a given direction from the center of the transmitting antenna. A solid angle of a cone inscribed on a sphere of radius R is given by the surface area of its

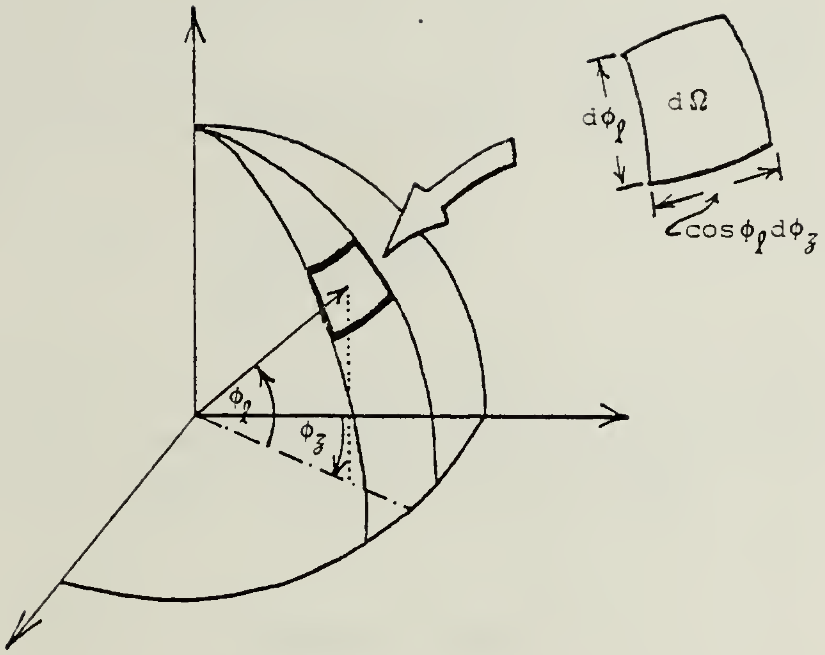


Figure 2.2 Antenna Geometry.

spherical cap divided by R^2 and is measured in units of steradians. The solid angle of an entire sphere of radius R is the spherical surface area ($4\pi R^2$) divided by R^2 which results in 4π steradians. Therefore an antenna pattern [$W(\phi_\ell,\phi_z)$] can be restated as the amount of power which will pass through a unit solid angle (1 steradian) in a given direction (ϕ_ℓ,ϕ_z) from the antenna center. The antenna pattern is usually normalized by comparing it to an isotropic antenna of equal transmitted power, to form the antenna gain function $g(\phi_\ell,\phi_z)$ and defined by

$$g(\phi_\ell,\phi_z) = \frac{W(\phi_\ell,\phi_z)}{\text{power per unit solid angle of an isotropic radiator with equal total transmitted power}} \quad \text{(eqn 2.2)}$$

An isotropic radiator is an antenna which radiates equal amounts of power in all directions. [Ref. 9].

Define P_T as the total power transmitted by an antenna. Then P_T can be determined by summing the power over all points on the unit sphere or, more conveniently, by integrating $W(\phi_\ell, \phi_z)$ over the unit sphere, that is,

$$P_T = \int_{\substack{\text{unit}\\ \text{sphere}}} W(\phi_\ell, \phi_z)\,d \qquad \text{(eqn 2.3)}$$

where $d\Omega$ represents the differential of the solid angle. It can be shown that the length of an arc inscribed on a circle of R radius by a radian angle θ is given by:

$$\text{length} = R\theta.$$

Then a change in elevation angle $(d\phi_\ell)$ results in an arc of length $d\phi_\ell$ on the surface of a unit sphere $(R=1)$. Additionally a change in the azimuth angle $(d\phi_z)$ results in an arc of a length that is dependent on the applied elevation angle such that:

$$\text{length} = \cos\phi_\ell\, d\phi_z$$

Then

$$d\Omega = \cos\phi_\ell\, d\phi_\ell\, d\phi_z$$

as depicted in Figure 2.2 . Therefore Equation 2.3 can be rewritten as

$$P_T = \int_{-\pi}^{\pi}\int_{-\pi/2}^{\pi/2} W(\phi_\ell, \phi_z)\cos\phi_\ell\, d\phi_\ell\, d\phi_z\ .$$

The total power per unit solid angle of an isotropic radiator with total power P_T is found by dividing the total power by the number of steradians contained within a sphere which is 4π. Thus Equation 2.2 can be written as:

$$g(\phi_\ell,\phi_z) = \frac{W(\phi_\ell,\phi_z)}{P_T\,/\,4\pi} \ . \qquad \text{(eqn 2.4)}$$

It can be seen that the gain function of a transmitting antenna is then simply a normalized version of its antenna pattern . Therefore, if the gain function is integrated over the unit sphere it yields the same value as $\frac{W(\phi_\ell,\phi_z)}{P_T\,/\,4\pi}$ integrated over the unit sphere such that:

$$\int_{\substack{\text{unit}\\ \text{sphere}}} g(\phi_\ell,\phi_z)\,d\Omega = \frac{W(\phi_\ell,\phi_z)}{P_T\,/\,4\pi}\,d\Omega \ . \qquad \text{(eqn 2.5)}$$

Now $W(\phi_\ell,\phi_z)$ integrated over the unit sphere yields the total transmitted power P_T so that

$$\int_{\substack{\text{unit}\\ \text{sphere}}} g(\phi_\ell,\phi_z)\,d = \frac{P_T}{P_T/4\pi}$$

or

$$\int_{\substack{\text{unit}\\ \text{sphere}}} g(\phi_\ell,\phi_z)\,d\Omega = 4\pi \ . \qquad \text{(eqn 2.6)}$$

Therefore, the volume under the gain function surface is a constant. This implies that if the gain function is increased in one direcion, it must be suitably decreased in other directions to maintain the constant integrated value of 4π. [Ref. 10].

Most transmitting antennas are designed to transmit their radiated field in a specified direction. Thus the gain function is usually peaked along what might be called a preferred direction and such a peak is referred to as the antenna main lobe. Transmissions in other directions are called sidelobes and usually represent power transmitted in unwanted directions. Figure 2.3 is a typical parabolic antenna pattern plotted as a function of azimuth angle in degrees, for the elevation plane $\phi_\ell=0$ and shows the mainlobe in the forward direction.

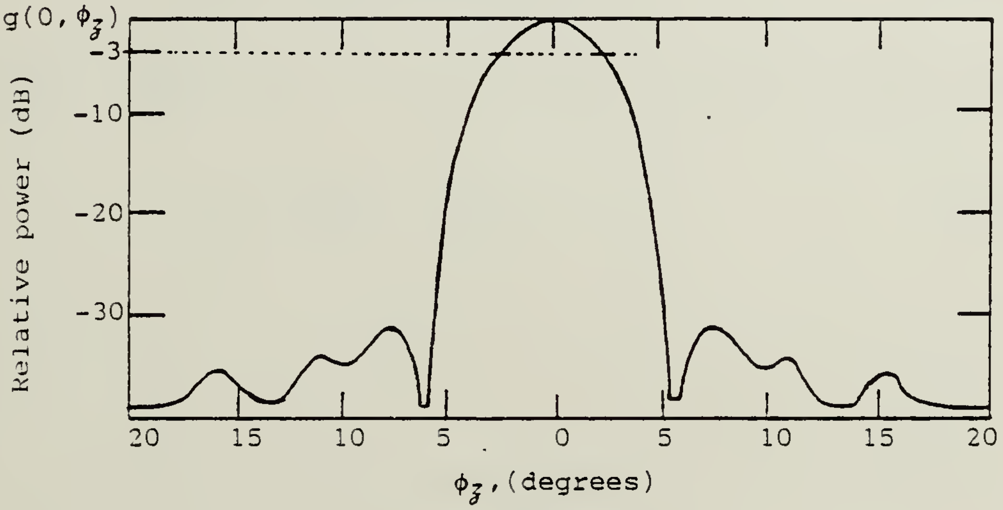


Figure 2.3 Parabolic Antenna Gain Function.

The actual antenna gain function is a cumbersome description of the antenna radiated field pattern; often simpler specifications are used such as antenna gain and field of view. The gain g of a transmitting antenna is defined as the maximum value of its gain function:

$$g \overset{\Delta}{=} \max g(\phi_\ell , \phi_z) \ , \qquad \text{(eqn 2.7)}$$

and is usually stated in decibels. The antenna field of view is a measure of the solid angle into which all of the transmitted field power is concentrated. In other words, the field of view is a measure of the directional properties of the antenna. For simplicity the field of view is defined as the solid angle (Ω_{FV}) through which all the radiated power would pass if the antenna pattern in this angle was constant and equal to its maximum value. Thus Figure 2.3 would be interpreted as shown in Figure 2.4. That is to say that $\Omega_{FV}[\max W(\phi_\ell, \phi_z)] = P_T$ or

$$\Omega_{FV} \overset{\Delta}{=} \frac{P}{\max W(\phi_\ell, \phi_z)} , \qquad \text{(eqn 2.8)}$$

However from Equations 2.4 and 2.7 it can be shown that

$$\Omega_{FV} = \frac{4\pi}{g} \text{steradians} . \qquad \text{(eqn 2.9)}$$

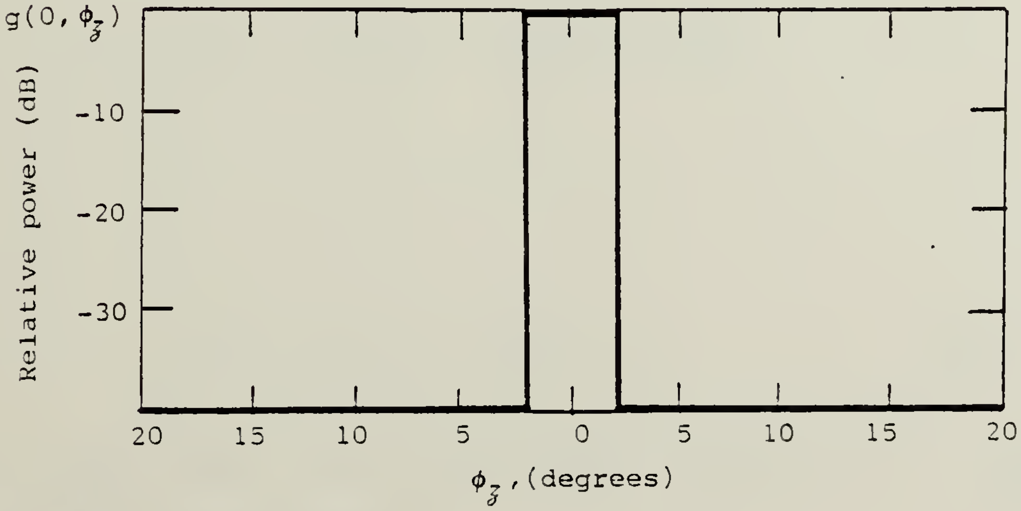


Figure 2.4 Ideal Antenna Gain Function.

Field of view is therefore inversely related to gain. This means that high gain antennas exhibit narrow fields of view. Rather than using the solid angle field of view, discussions and specifications regarding antennas can be more easily dealt with in terms of the planar angle beamwidths which are given in terms of radians or degrees and are defined separately for azimuth and elevation planes. For example, the antenna pattern in Figure 2.2 has a main lobe azimuth beamwidth of 4° measured at the 3dB points. For a symmetric gain pattern which is obtained by rotating the azimuth gain pattern about the azimuth axis, the planar beamwidth (designated ϕ_b and given in radians) is related to the solid angle field of view (in steriadians) by

$$\Omega_{FV} = 2\pi[1-\cos(\phi_b/2)]$$

which, using the trigonometric identity ($\cos 2a=1-2\sin^2 a$) can be written as

$$\Omega_{FV} = 4\pi\left\{\tfrac{1}{2}[1-\cos(\phi_b/2)]\right\} .$$

Then

$$\Omega_{FV} = 4\pi[\sin^2(\phi_b/4)] . \qquad \text{(eqn 2.10)}$$

It can be shown that for very small values of x, $\sin(x) \approx x$. Thus where ϕ_b is very small

$$\Omega_{FV} \approx 4\pi(\phi_b/4)^2$$

Then

$$\Omega_{FV} = \pi \, \phi_b^2/4 \; .$$

Therefore, when given a transmitting antenna's field of view, the planar beamwidth ϕ_b can be determined (in radians) by the equation

$$\phi_b = \sqrt{\frac{4}{\pi} \, \Omega_{FV}} \qquad \text{(eqn 2.11)}$$

provided it is known that ϕ_b is small and the spatial gain pattern is symmetrical. [Ref. 11].

No transmitting antenna is an ideal, loss free radiator. Therefore, the power P_T radiated by it is less than the power fed into the antenna input terminals. It then becomes convenient to define an effective antenna transmitting gain relative to the power actually coupled into the antenna input terminals. The power fed into the input terminals of the antenna is denoted P_t. Thus the effective gain function is defined as

$$\tilde{g}(\phi_\ell, \phi_z) = \frac{W(\phi_\ell, \phi_z)}{P_T \, / \, 4} \; . \qquad \text{(eqn 2.12)}$$

This means

$$\tilde{g}(\phi_\ell, \phi_z) = \rho_r \; g(\phi_\ell, \phi_z) \qquad \text{(eqn 2.13)}$$

where $\rho_r = P_T / P_t$ is the antenna radiation efficiency factor and, in essence, is a measure of the radiation losses of an antenna. It should be noted that $\rho_r \leq 1$, so that the effective antenna gain $\tilde{g}$ is always less than the gain g of the

antenna pattern. Typical radiation efficiencies are on the order of $0.90 \leq \rho_r \leq 0.99$. corresponding to a reduction of about 0.5dB in gain. [Ref. 12].

The antenna gain function permits simplified calculations of the amount of transmitted field power that will impinge on a perpendicular (normal) receiving surface of area A, located a distance D in the direction (ϕ_{ℓ}, ϕ_{z}) from the transmitting antenna (Figure 2.5). Letting P_A represent the power impinging on area A, Equation 2.1 can be employed so that

$$P_A = W(\phi_{\ell_o}, \phi_{z_o})\Omega_a$$

which states that the amount of power P_A is equal to the amount of power transmitted per unit solid angle in the direction $(\phi_{\ell_o}, \phi_{z_o})$ multiplied by Ω_d solid angle that subtends the surface of area A.

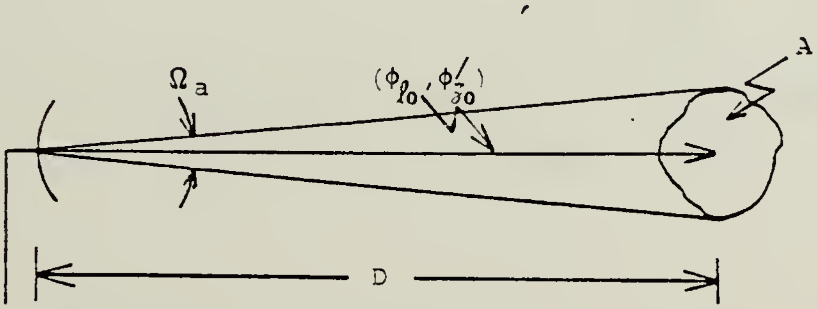


Figure 2.5 Antenna Propagation.

When the distance D is much larger than the largest dimension of the surface of area A, then the latter approximates a section of a spherical surface and total solid angle subtended by the surface area A is equal to A divided by the

distance squared, that is,

$$\Omega_a \approx A/D^2 .$$

Therefore

$$P_A \approx W(\phi_{\ell_0}, \phi_{z_0}) A/D^2 .$$

Using Equations 2.12 and 2.13 it can be seen that

$$P_a \approx \left[\frac{P_t \rho_r g(\phi_{\ell_0}, \phi_{z_0})}{4\pi D^2} \right] A . \qquad \text{(eqn 2.14)}$$

The quantity in brackets has units of power per area and thus represents the power density, called the field intensity or flux density of the propagating electromagnetic field at the distance D in direction $(\phi_{\ell_0}, \phi_{z_0})$. The numerator is often referred to as the effective isotropic radiated power (EIRP) in the direction $(\phi_{\ell_0}, \phi_{z_0})$ and represents the equivalent power which an isotropic antenna must transmit in order to achieve the same field power density at a distance D from the antenna. Then, EIRP can be written as:

$$\text{EIRP} = P_t \rho_r g(\phi_{\ell_0}, \phi_{z_0}) . \qquad \text{(eqn 2.15)}$$

It should be noted from Equation 2.14 that when propagating in free space, the electromagnetic field intensity decreases as a function of the distance squared. [Ref. 13].

An intrinsic property of any antenna is that its transmitting field of view at a signal wavelength λ is related to

the physical area of the antenna A by

$$\Omega_{FV} = \frac{\lambda^2}{\rho_{ap}\ A_a} \qquad \text{(eqn 2.16)}$$

where ρ_{ap} is the antenna aperture loss factor. This aperture loss factor accounts for antenna diffraction losses with values ranging from 0.5 to 0.75. Therefore, substituting Equation 2.16 into Equation 2.9 means that the antenna gain is given by

$$g = \left(\frac{4\pi}{\lambda^2}\right)\rho_{ap}\ A_a\ . \qquad \text{(eqn 2.17)}$$

Wavelength is related to frequency (f) by the equation $c = \lambda f$ where c is the velocity of an electromagnetic wave traveling in a vacuum and has a constant value of 3×10^8 meters per second. Then

$$g = 4\pi\left(\frac{f}{c}\right)^2 \rho_{ap}\ A_a\ . \qquad \text{(eqn 2.18)}$$

It is now readily apparent that for a fixed antenna size, as the operating frequency increases the antenna's gain increases and its field of view narrows. Alternately, for a specific carrier frequency, higher gains and narrower field of views require larger antennas. [Ref. 14].

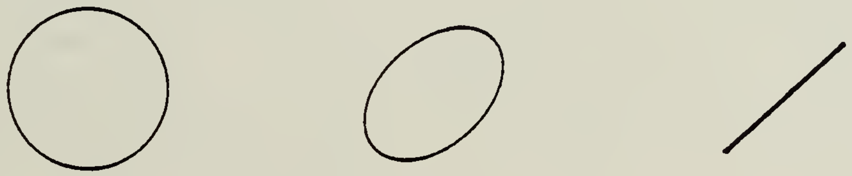
E. RECEIVING ANTENNAS

In the previous section it was shown that transmitting antennas convert electronic signals to electromagnetic fields and radiate them as power which decreases in field intensity as the distance from the antennas increases. Similarly, a receiving antenna converts impinging electromagnetic fields to electronic signals whose power is

proportional to the intensity of the field power intercepted by the receiving antenna. Like transmitting antennas, receiving antennas have a fixed coordinate system with the origin located at the center of the antenna. Unlike transmitting antennas which are considered point sources, receiving antennas are described in terms of an effective area $A(\phi_\ell, \phi_z)$ which is a function of the direction of arrival (ϕ_ℓ, ϕ_z) of the plane wave, referenced to the antenna's coordinate system. The effective receiving antenna area in the direction (ϕ_ℓ, ϕ_z) is then defined as [Ref. 15]:

$$A(\phi_\ell, \phi_z) = \frac{\text{power observed due to the field from the direction } (\phi_\ell, \phi_z)}{\text{power density of the field}} \; . \qquad \text{(eqn 2.19)}$$

As Figure 2.6 illustrates, the antenna area is a function of the direction angles (ϕ_ℓ, ϕ_z). Additionally, the antenna area includes associated antenna losses and polarization mismatches with respect to the electromagnetic field.



a) $(0^\circ, 0^\circ)$ b) $(45^\circ, 45^\circ)$ c) $(45^\circ, 90^\circ)$

Figure 2.6 Circular Receiving Antenna Geometry.

The effective area therefore describes a receiving pattern over all directions from the antenna coordinates. The maximum area

$$A_m = \max_{\phi_\ell, \phi_z} A(\phi_\ell, \phi_z) \qquad \text{(eqn 2.20)}$$

is the largest area presented to arriving fields. This maximum area is less than the physical antenna area A_a due to the receiving aperture loss factor ρ_{ap}. This aperture loss factor is due to diffraction, and

$$A_m = \rho_{ap} A_p \quad . \qquad \text{(eqn 2.21)}$$

From the Rayleigh-Carson theorem of transmitting and receiving plane wave antennas [Ref. 16], it can be shown that

$$\frac{A(\phi_{\ell},\phi_{z})}{A_m} = \frac{g(\phi_{\ell},\phi_{z})}{g} \qquad \text{(eqn 2.22)}$$

where $g(\phi_{\ell}, \phi_{z})$ is the antenna gain function that would result if the antenna were used to transmit signals. In general it can then be said that, except for scaling factors, and antenna's transmitting and receiving patterns are the same, and the ability of an antenna to receive from a given direction equals its ability to transmit power in that same direction. From Equation 2.17, where A_a is the antenna physical area, and the reciprocity principle it can be seen that $A_a = A_p$ so that

$$A_m = g \frac{\lambda^2}{4\pi} \frac{\rho_{ap}}{\rho_{ap}} = g \frac{\lambda^2}{4\pi}$$

or rewritten in the form

$$\frac{A_m}{g} = \frac{\lambda^2}{4\pi} \qquad \text{(eqn 2.23)}$$

and substituting into Equation 2.22

$$A(\phi_\ell,\phi_z) = \frac{\lambda^2}{4\pi}\, g(\phi_\ell,\phi_z) \quad . \qquad \text{(eqn 2.24)}$$

This then relates antenna receiving area pattern directly to the gain pattern and the operating frequency. Furthermore, the difference between the receiving antenna area function and the antenna gain function is $\lambda^2/4\pi$. Now recalling Equation 2.9 and substituting according to Equation 2.23, $\Omega_{FV} = \lambda^2/A_m$. But from Equation 2.21 $A_m = \rho_{ap} A_p$, so that

$$\Omega_{FV} = \frac{\lambda^2}{\rho_{ap} A_p} \qquad \text{(eqn 2.25)}$$

which is the solid angle associated with maximal power reception of the receiving antenna. This then implies that directionality requires high operating frequencies in order to obtain antennas of reasonable size. Thus the design of receiving antennas can be equivalently stated in terms of transmitting antenna gain patterns. Therefore, if a receiving antenna is required to have a given field of view it can be designed as if it were a transmitting antenna with the same field of view. For example, an antenna with the pattern shown in Figure 2.3 would collect transmitted power primarily over a 4° beamwidth as measured from the 3dB points. [Ref. 17].

C. RECEIVED POWER

The actual amount of carrier power that can be transmitted to a receiver is a key parameter in assessing communication system performance and controls management decisions regarding equipment specifications, link distances, operating frequencies, and construction costs.

Figure 2.7 is a typical communication system which operates at a fixed carrier frequency f_c. Within the transmitter, the information signal passes through a transmitter power amplifier which produces a modulated carrier signal with carrier power P_{amp}. The carrier power is the output of the transmitter and is coupled to the transmitting antenna terminals by means of either a waveguide or a cable. The power coupled to the antenna is given by

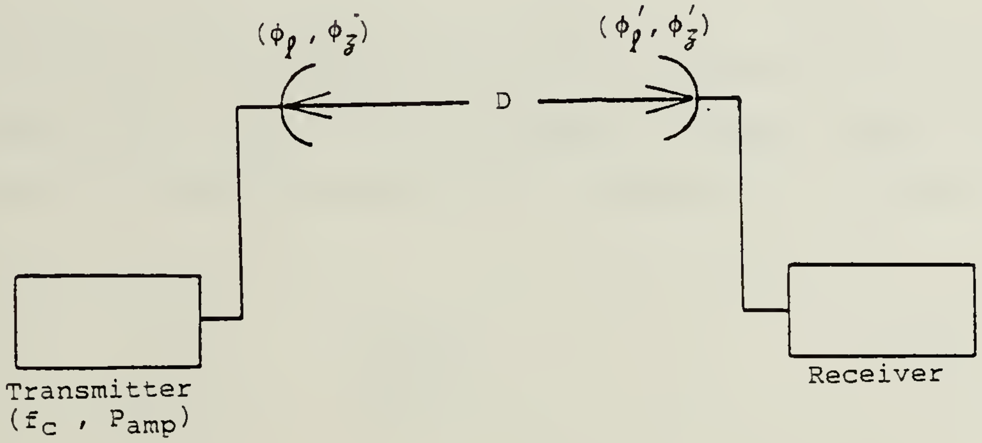


Figure 2.7 Transmitting and Receiving Channel.

$$P_t = P_{amp} L_T \qquad \text{(eqn 2.26)}$$

where L_T account for the guide or coupling losses at the transmitter. From Equation 2.15 the EIRP of the transmitter in the direction of the receiver is

$$EIRP = P_t \rho_r g_t(\phi_\ell, \phi_z) \qquad \text{(eqn 2.27)}$$

where ρ_r is the radiation loss factor of the transmitting antenna and $g(\phi_\ell, \phi_z)$ is the transmitting gain function in the direction of the receiving antenna, located some

distance (D) away. As demonstrated by Equation 2.14, the power density P_{den} arriving at the receiving antenna after propagating a distance D is given by

$$P_{den} = \frac{EIRP}{4\pi D^2} \quad . \qquad \text{(eqn 2.28)}$$

Assuming that propagation of the plane wave is through a medium other than a perfect vacuum, an additional loss (L_a) may be experienced due to atmospheric absortion, rainfall, etc. Thus the power density is reduced by a factor L_a such that

$$P_{den} = \frac{EIRP}{4\pi D^2} L_a \quad . \qquad \text{(eqn 2.29)}$$

It should be noted that for free space $L_a = 1$ which equates to no loss. The power collected by the receiving antenna presenting an effective area $A(\phi'_\ell , \phi'_z)$ to the arriving plane wave is

$$P_{ant} = P_{den} A(\phi'_\ell , \phi'_z)$$

and from Equation 2.24 for $A(\phi'_\ell , \phi'_z)$ it can be seen that

$$P_{ant} = P_{den} \frac{\lambda^2}{4\pi} g(\phi'_\ell , \phi'_z) \quad . \qquad \text{(eqn 2.30)}$$

During conversion from the electromagnetic field to an electronic signal, a portion of the intercepted power is lost and accounted for by introducing a loss factor L_r. Thus received power P_r is

$$P_r = P_{ant} L_r$$

and from Equations 2.26, 2.27, 2.29, and 2.30 it can be shown that total collected carrier power at the output terminal of the receiving antenna is given by

$$P_r = P_{amp} \left(\frac{L_t L_a \rho_r}{4\pi D^2} \right) g_t(\phi_\ell, \phi_z) \frac{\lambda^2}{4\pi} g_r(\phi_\ell', \phi_z') \; L_r \; . \qquad \text{(eqn 2.31)}$$

It becomes convenient to interpret

$$L_p \triangleq \frac{1}{4\pi D^2} \; \frac{\lambda^2}{4\pi} = \left(\frac{\lambda}{4\pi D} \right)^2 \qquad \text{(eqn 2.32)}$$

as free space propagation loss so that Equation 2.31 can be written as

$$P_r = P_{amp} \rho_r L_t L_a L_p L_r g_t(\phi_\ell, \phi_z) g_r(\phi_\ell', \phi_z') \; .$$

Assuming the antennas are properly aligned $(\phi_\ell, \phi_z = \phi_\ell', \phi_z')$ so that the transmitter and receiver directions are at the peak of the gain functions, then P_r can be written as

$$P_r = P_{amp} \rho_r L_t L_a L_p L_r g_t g_r \qquad \text{(eqn 2.33)}$$

where g_t and g_r are now the respective antenna gains. This equation then is a summary of the effect of the complete carrier transmission subsystem as the power flows from the transmitter power amplifier to the receiving antenna terminals.

It can be seen that Equation 2.33 involves simply a product of the gains and losses along the signal path. Usually, gains and losses are given in terms of dB. Therefore, it is convenient to express the equation for P_r

in dB notation. This results in

$$(P_r)_{dB} = \begin{matrix} (P_{amp})_{dB} & +(\;_r)_{dB} & +(L_t)_{dB} & +(L_a)_{dB} \\ +(L_p)_{dB} & +(L_r)_{dB} & +(g_t)_{dB} & +(g_r)_{dB} \end{matrix} \qquad \text{(eqn 2.34)}$$

where all factors are dB values with the loss factors being negative. The propagation loss factor L_p when put in dB notation reduces to either

$$(L_p)_{dB} = [-36.6 - 20\log D - 20\log(f_c)]_{dB} \qquad \text{(eqn 2.35)}$$

when distance is in miles and frequency is in MHz, or

$$(L_p)_{dB} = [-32.4 - 20\log D - 20\log(f_c)]_{dB} \qquad \text{(eqn 2.36)}$$

when distance is in kilometers and frequency is in MHz. The final result is that the received power available from the receiving antenna is given by Equation 2.34. This equation is known as the power budget equation. It must be used if one is to analyze system performance in terms of received power levels.

Example 2.1: Assume a communications system must be designed for placement in mountainous terrain where the transmitting and receiving stations are to be located approximately 30 miles apart. The frequency manager for the area has assigned 270MHz as the carrier frequency. If the output of the transmitter power amplifier is 100watts, guide and coupling losses to the transmitting antenna total 1dB, and the radiation loss factor of the transmitting antenna is 0.95, then the total radiated power is 18.777dBW. The propagation loss can be calculated to be 114.74 dB. If the total atmospheric losses due to absorption and scattering are 3dB, the guide and coupling losses of the receiver's antenna

section are identical to the transmitter's, and the minimum detectable signal level is 1μwatt, it can be determined that the combined gain of the two antennas must be 40dB. Assuming that the antennas employed are identical dish antennas with an aperture loss factor of 0.75, their size can be determined to be approximately 1 foot in diameter. Thus the cost of setting up such a system should be moderate.

D. ANTENNA PROPAGATION CHANNELS

There are four basic propagation channels that can be defined for the electromagnetic fields transmitted by antennas, as indicated in Figure 2.8. These are (1) the ground wave channel; (2) the space wave channel; (3) the sky wave channel; and (4) the outer space wave channel. Each is appropriate in a specific application and each defines a slightly different propagation model. [Ref. 18].

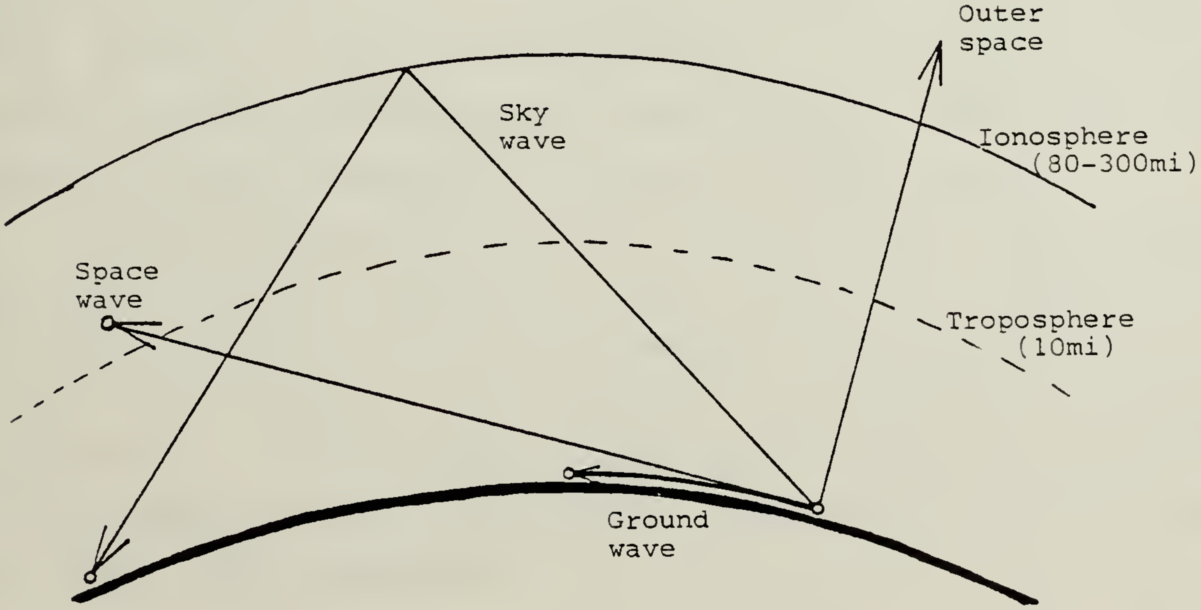


Figure 2.8 Free Space Propagation Channels.

1. Ground Wave Channel

When both the transmitter and receiver are located within a few meters of the earth, the electromagnetic field behaves as a wave propagating within a waveguide with the earth itself serving as a waveguide wall. Such models are applicable in landbased mobile and hand-held communication systems. The ground wave channel is characterized by strong attenuation with distance and attenuation values increasing sharply as carrier frequency increases. Thus the ground wave channel is practical only for systems operating at or below the HF range over distances of a few miles. [Ref. 19].

2. Sky Wave Channel

The sky wave channel results when the propagating electromagnetic field is reflected by either the ionosphere or troposphere boundary region. The ionosphere is a region of free electrons trapped in a belt around the earth. To the longer wavelength carrier frequencies (HF and below) the ionosphere appears as a conducting surface which reflects carrier energy. This characteristic allows long range, over-the-horizon communications such as the Navy's HF ship-to-shore or fleet broadcast channels. [Ref. 20].

As has been pointed out, antenna size is inversely related to carrier frequency (the lower the frequency the larger the antenna). For this reason, frequencies below the HF range are usually not employed for sky wave channel transmission. On the other hand, the smaller wavelengths above HF penetrate the ionosphere and are absorbed or scattered rather than reflected.

3. Space Wave Channel

The space wave channel is strictly a line-of-sight (LOS) channel contained within the earth's atmosphere whereby the electromagnetic field propagates directly from the transmitter to the receiver. Typical examples of such communication channel use are earth-airplane, airplane-airplane, and microwave relay systems. Since LOS systems are fitted on highly mobile platforms or placed on fixed site structures which require raising antennas high enough to preclude severe signal ground attenuation, frequencies of operation are restricted to that portion of the spectrum above HF. [Ref. 21].

Maximum reception distance or the LOS distance of systems utilizing the space wave channel is dependent on the altitude of both the transmitting and receiving antennas as shown in Figure 2.9. Clearly, if the height of the antenna at station 2 and 3 in Figure 2.9 is equal, then as the height of the antenna at station 1 increases, so does the line-of-sight distance ($D_3 > D_2$). Referring to Figure 2.10

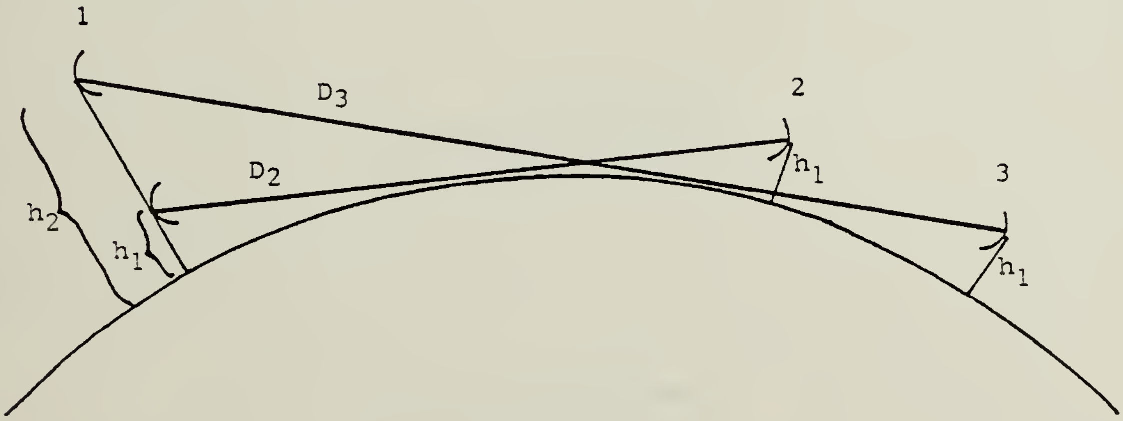


Figure 2.9 LOS Geometry.

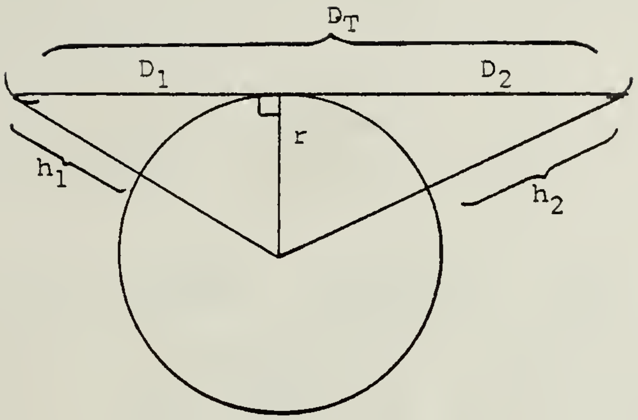


Figure 2.10 LOS Determination.

the LOS distance can be determined using Pythagorean relations. The distance D_1 can be found by

$$(D_1)^2 + r^2 = (h_1+r)^2 \qquad \text{(eqn 2.37)}$$

where r is the earth's radius and h_1 is the height of one antenna. Then

$$D_1 = \sqrt{h_1^2 + 2h_1 r}\ .$$

Likewise

$$D_2 = \sqrt{h_2^2 + 2h_2 r}\ .$$

Thus

$$D_T = D_1 + D_2$$
$$D_T = \sqrt{h_1^2 + 2h_1 r} + \sqrt{h_2^2 + 2h_2 r}$$

Now $h_1 << r$ and $h_2 << r$ so that $h_1^2 << 2h_1r$ and $h_2^2 << 2h_2r$. Therefore by approximation

$$D_T = \sqrt{2h_1r} + \sqrt{2h_2r} \qquad \text{(eqn 2.38)}$$

Furthermore, the average radius of the earth (r) is 3960miles whereas the heights h_1 and h_2 are usually expressed in feet. Then Equation 2.38 yields the approximate LOS distance in miles for two antennas elevated h_1 and h_2 feet. If either antenna is located at ground level the respective term simply is 0. So from Equation 2.38

$$LOS_{(miles)} = 1.2(\sqrt{h_1} + \sqrt{h_2}) \qquad \text{(eqn 2.39)}$$

This then is the distance used to calculate the propagation path loss in the power budget equation (for a transmission set up similar to Figure 2.9).

Since the space wave channel is contained within the earth's atmosphere, the propagating wave is also subjected to attenuation due to atmospheric particles. This effect is particularly severe when the carrier frequency/wavelength approaches the size of the various particles in the atmosphere. Rainfall tends to be a particularly significant attenuator since water drop sizes approach the wavelengths associated with the higher microwave frequencies.

4. Outer Space Channel

An outer space channel involves transmission out through the earth's atmosphere to satellites and space stations. In this channel energy must pass through the ionosphere boundary layer rather than be reflected. Thus outer space channels require use of operating frequencies well above the HF band. As the carrier frequency is

increased into the VHF and SHF band a larger portion of the field energy passes through the ionosphere rather than being scattered and reflected. The signal must still pass through the atmosphere and is subject to atmospheric attenuation; however, the relatively small distance of atmosphere compared to an extraordinarily long propagation distance in a near vacuum significantly reduces the impact of the atmospheric effects. Thus the major attenuating factor for the outer space channel is propagation path loss. Furthermore, satellite to satellite or space-station to space-station communications would have no atmospheric losses since communications would be totally outside the earth's atmosphere. [Ref. 22].

5. ELF Channel

One final channel which has not been previously identified is the ELF channel. It is only mentioned because of recent work conducted in the ELF band and the approved Department of Defense's ELF project as part of the strategic triad. The principle underlying the propagation of an ELF carrier signal is that at the ELF frequencies the earth and ionosphere boundary layer act like the interior walls of a waveguide. Waveguides are normally constructed such that the principal dimension is either 1/4 or 1/8 of the carrier frequency wavelength. Thus if the boundary of the ionosphere is 300 miles above the earth, the operating frequency can be determined by

$$(\lambda/8) = 300\text{miles}.$$

where $\lambda=c/f$. So that $f=(c/\lambda)=77\text{Hz}$. It can therefore be projected that ELF will be operated at or near 77Hz and/or 155Hz. [Ref. 23].

E. COMMUNICATION SATELLITES

To be of value as a relay point for communications from one point on Earth to another, a communications satellite must have a predictable orbit. The orbit must also have characteristics which permit reasonable terminal designs that can be affordably deployed at both fixed sites and aboard mobile platforms. A number of satellite orbits have value for different communications systems. By far the most popular is the geosynchronous orbit satellite which has the unique characteristic of appearing fixed in space relative to an Earth observer.

1. Orbital Parameters

Three fundamental laws govern the orbital motion of Earth satellites. These are:

1. Newton's law of universal gravity - The force on the satellite due to the earth's gravitational field is inversely proportional to the distance between the earth and satellite centers.
2. Kepler's first law - Satellite orbits lie in planes which pass through the earth's center and are conic sections.
3. Kepler's second law - Elliptical orbit satellites have orbit period (T) and orbit semi-major axis (2a) which are uniquely related as $T \propto a^{1.5}$.

Figure 2.11 shows two conic sections of interest with respect to communication satellites. These are the ellipse and the special case ellipse which is a circle. Figure 2.12 shows the general ellipical orbit. The major axis is 2a and the minor axis is 2b. The central body of the orbit is the

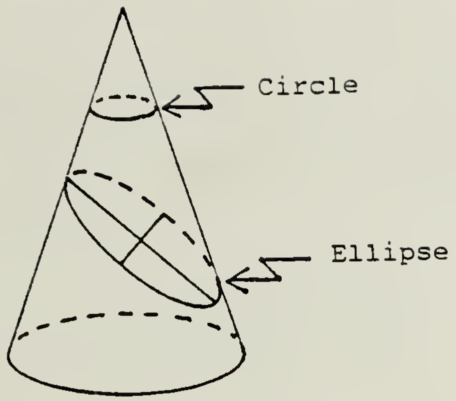


Figure 2.11 Conic Sections.

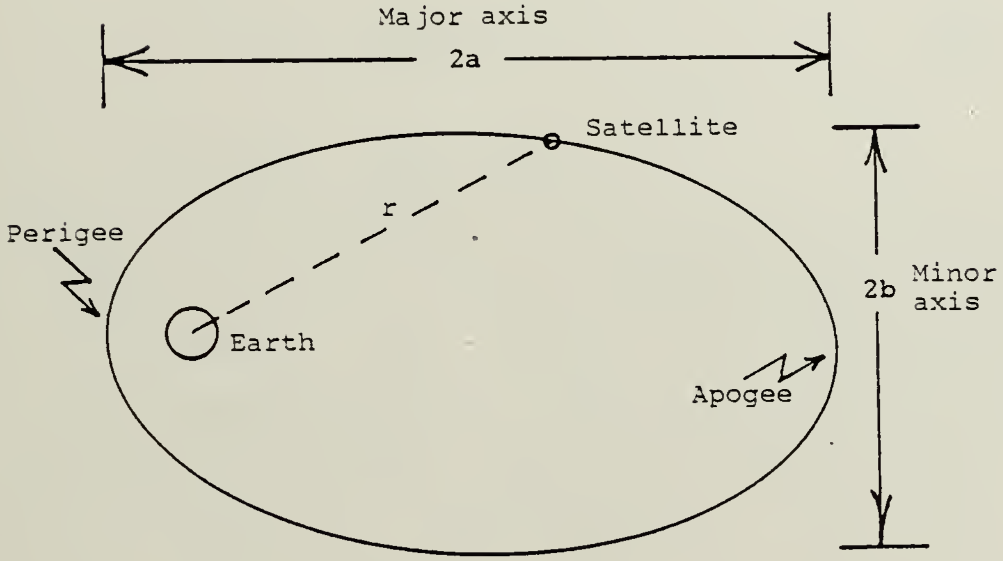


Figure 2.12 Elliptical Orbit.

Earth. The closest point of approach to Earth is called perigee and the farthest distance from Earth is called apogee. A characteristic of an elliptical orbit which is of some interest is eccentricity of the orbit, e. Eccentricity is a measure of an orbit's elliptical shape and is determined by $e=\sqrt{1-(b/a)^2}$. Thus as an orbit becomes less

elliptical, b/a increases so that e becomes smaller and smaller. For the special case of a circular orbit, b=a which is the radius of a circle and e=0. Two additional parameters of interest are the instantaneous/linear velocity of a satellite and its orbital period. Instantaneous velocity (v) is given by:

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} \qquad \text{(eqn 2.40)}$$

where μ is Earth's gravitational constant ($3.992 \times 10^5 km^3/sec^2$), r is the instantaneous distance to the satellite, and "a" is the semi-major axis of the orbit. The period (T) of an orbiting satellite is given by:

$$T = 2\pi a^{1.5}/\mu^{\frac{1}{2}} \qquad \text{(eqn 2.41)}$$

where T is in seconds. [Ref. 24].

The one orbit which is of importance to present day satellite communications is the geosynchronous orbit. As previously mentioned, this orbit is characterized by a circular orbit (eccentricity of zero) with period and inclination selected so that the satellite appears stationary to an observer on Earth. Inclination is the tilt of the orbital plane with respect to Earth's equatorial plane as illustrated in Figure 2.13. Thus it can be seen that for a geosynchrous orbit, the satellite must be positioned above the earth's equator. Because the earth is in orbit around the sun in addition to rotating about its north-south axis, the orbital period of a geosynchronous satellite can not be determined as simply as its inclination. As it turns out, the orbital period of a geosynchronous satellite is one sidereal day or T = 86,164 seconds. [Ref. 25].

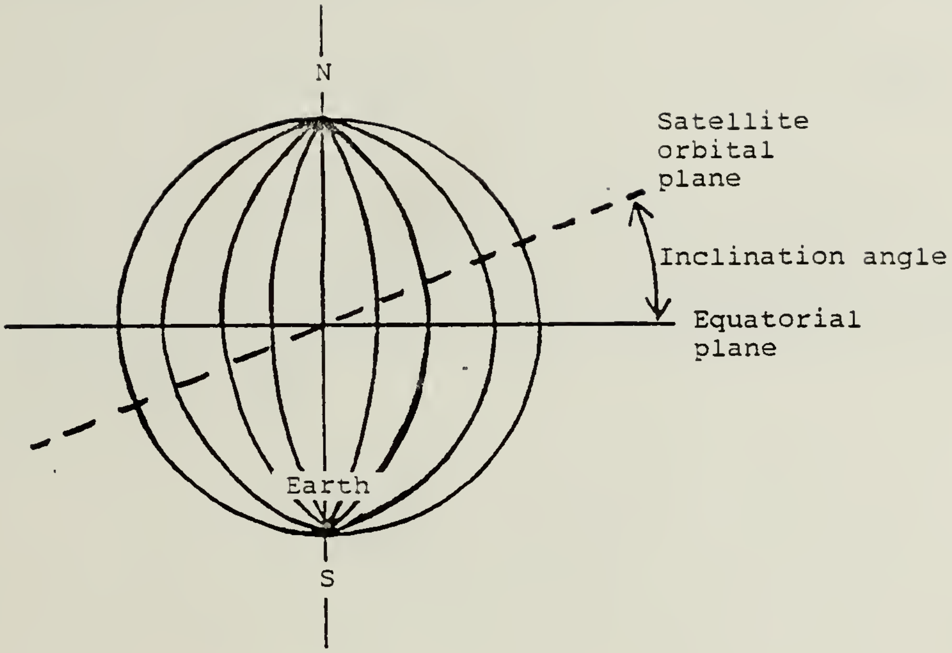


Figure 2.13 Orbit Inclination.

The sub-orbital point of a geosynchronous satellite is that point on Earth directly beneath the satellite. The distance from this point to the satellite can be determined by studying the forces acting on the satellite. When the satellite is in its orbit, the gravitational force between the satellite and Earth is balanced by the centrifugal force of the satellite orbiting Earth. From physics, the gravitational force is given by

$$F = G\,\frac{M_e M_s}{r^2} \qquad \text{(eqn 2.42)}$$

where G is the universal gravitational constant ($6.673\times10^{8}\,cm^3/gm\text{-}sec^2$) M_e is the mass of Earth ($5.983\times10^{23}\,gm$), M_s is the mass of the satellite , and r is the distance between the centers of Earth and the satellite.

The centrifugal force of the orbiting satellite is:

$$F = M_s \omega^2 r \qquad \text{(eqn 2.43)}$$

where ω is the angular velocity of the satellite and is equal to $2\pi/T$. When equilibrium is achieved, the forces described by Equations 2.42 and 2.43 are equal and opposite. Thus it can be seen that:

$$G \frac{M_s M_e}{r^2} = M_s \omega^2 r$$

so that

$$r = \left(\frac{G\ M_e}{\omega^2} \right)^{1/3} . \qquad \text{(eqn 2.44)}$$

Now G M_e is simply the earth's gravitational constant μ,, and $\omega = 2\pi/T$. Therefore Equation 2.43 can be written in the form

$$r = \left(\frac{\mu T^2}{4\pi^2} \right)^{1/3} . \qquad \text{(eqn 2.45)}$$

Substituting the appropriate values into the equation for μ and T results in an orbital radius 42,185Km (26,212miles) from the center of Earth. The sub-orbital distance is the satellite orbital radius minus Earth's equatorial radius (6378Km/3963miles). The sub-orbital radius is 35,807Km (22,249miles). Thus a communication signal transmitted between the satellite and a sub-orbital Earth station would have a propagation path loss applied over a 22,249 mile link.

The tangential velocity required for a geosynchronous satellite to maintain orbit can be quickly determined by applying the results of Equation 2.45 to Equation 2.40. However, Equation 2.39 can be somewhat simplified since the semi-major axis of a circular orbit is equal to the instantaneous distance r from the earth which is the sub-orbital distance. Thus $V = \sqrt{\mu/r}$, so that the tangential velocity of a geosynchronous satellite is 11,074 Km/hr.

2. Antenna Considerations

If a satellite is to be used as a relay of communication signals between various points on Earth, the coverage or portion of the Earth which can be viewed from the satellite is important. Therefore, the satellite antenna beamwidth or field of view required to provide maximum Earth coverage must be determined. From a management perspective, the desired antenna beamwidth is such that the transmitted electromagnetic field impinges on the Earth's surface with no spill-over to the space beyond. Figure 2.14 illustrates the geometry to be employed to quantify the satellite antenna beamwidth necessary to provide maximum Earth coverage while conserving transmitter power output. In Figure 2.14, R is the earth's equatorial radius, h is the suborbital distance, θ_m is the maximum latitude of reception, and ϕ_m is the maximum coverage beamwidth. ϕ_m can be determined from

$$\frac{R}{R+h} = \sin(\phi_m/2) . \qquad \text{(eqn 2.46)}$$

Therefore,

$$\phi_m = 2\arcsin[R/(R+h)].$$

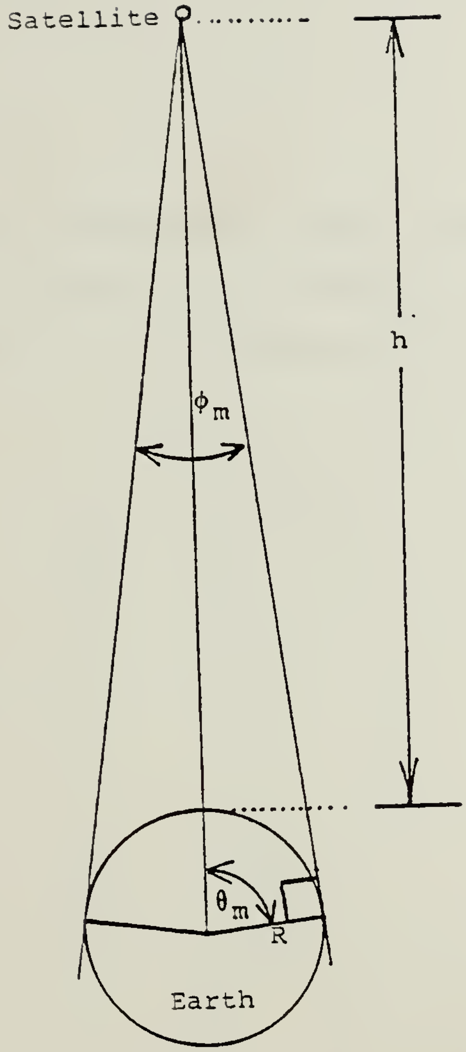


Figure 2.14 Maximum Satellite Coverage.

From the determination of the suborbital distance for geosynchronous satellites it can be shown that the beamwidth needed to provide maximum Earth coverage is 17.4°. Similarly, the maximum Earth coverage achieved with this antenna is given by:

$$2\theta_m = 2\arccos[R/(R+h)]$$

which turns out to be 162.6°. The maximum physical user

separation on Earth can be calculated from

$$s = r\theta \quad \text{(eqn 2.47)}$$

where s is the linear distance over a curved surface, r is the radius of the earth, and $\theta = 2\theta_m$ radians. Thus the maximum user separation is 11,246miles or 18,100Km.

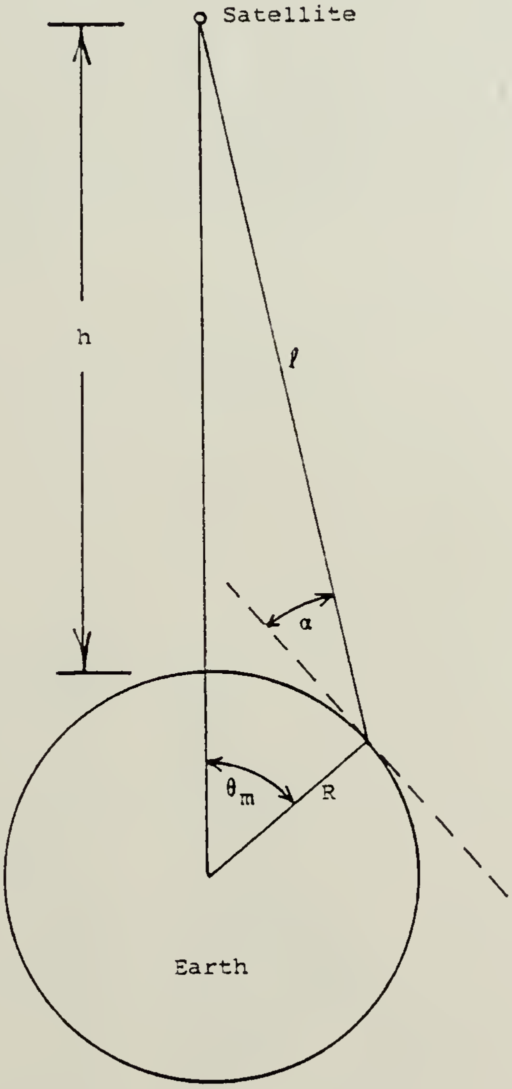


Figure 2.15 Earth Station Geometry.

For Earth stations which are not located at the suborbital point, two factors must be considered. These are the direct path distance to the satellite and the Earth station's antenna elevation angle to the satellite. Figure 2.15 illustrates the geometry used to determine these two factors. Since the geosynchronous satellite is positioned over the equator, the Earth station's latitude θ_m is directly related to the antenna elevation angle α and the direct path distance ℓ to the satellite. The triangle in Figure 2.15 can be broken into two distinct right triangles (as in Figure 2.16) to determine both α and ℓ. To determine ℓ the dimensions of the lower triangle must first be found and then applied to the second triangle. In turn this information can be used to solve for α. Knowing R and θ_m, A and B are found from

$$A = R\cos(\theta_m) \qquad \text{(eqn 2.48)}$$

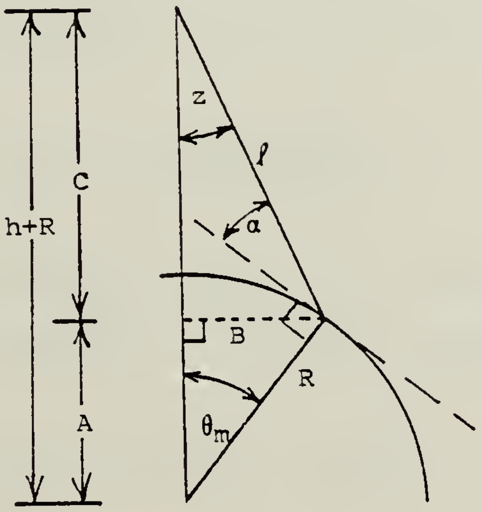


Figure 2.16 Partitioned Geometry.

and

$$B = R\sin(\theta_m)\,. \qquad \text{(eqn 2.49)}$$

Then

$$C = h+R-A = h+R[1-\cos(\theta_m)]$$

so that $\ell = \sqrt{B^2+C^2}$, or

$$\ell = (h^2+2hR+2R^2-2hR\cos\theta_m-2R^2\cos\theta_m)^{\frac{1}{2}} \qquad \text{(eqn 2.50)}$$

and the angle z is determined from $\tan(z)=B/C$. Or

$$z = \text{acrtan}\,\frac{R\sin(\theta_m)}{h+R-R\cos(\theta_m)}\,.$$

Now since there are a total of 180° in a triangle and α is the angle above the horizon then $\alpha=(180°-\theta_m-z)-90°$, or

$$\alpha = 90°-\theta_m-\arctan\frac{R\sin(\theta_m)}{h+R-R\cos(\theta_m)}\,. \qquad \text{(eqn 2.51)}$$

However, if the elevation angle is known then the generalized form of the Pythagorean theorem, namely

$$c^2 = a^2+b^2-2ab\cos\theta \qquad \text{(eqn 2.52)}$$

can be applied to Figure 2.15. It can then be shown that

$$(h+R)^2 = R^2+\ell^2-2R\ell\cos(90°+\alpha)\,.$$

But

$$\cos(90^\circ+\alpha) = -\sin(\alpha).$$

Thus $(h+R)^2$ can be rewritten and put into the quadratic form

$$\ell^2 + (2R\sin\alpha)\ell - (h^2+2hR) = 0 \qquad \text{(eqn 2.53)}$$

and, using the solution of a quadratic equation, ℓ is evaluated from

$$\ell = \frac{-2R\sin\alpha \pm \sqrt{4R^2\sin^2\alpha + 4h^2 + 8hR}}{2}. \qquad \text{(eqn 2.54)}$$

Now $0^\circ \leq \alpha \leq 90^\circ$. Furthermore, the distance ℓ must always be positive, therefore Equation 2.54 can be written as

$$\ell = -R\sin\alpha + \sqrt{R^2\sin^2\alpha + h^2 + 2hR} \qquad \text{(eqn 2.55)}$$

Example 2.2: Assume a geosynchronous communication satellite is "parked" at 177° East longitude. If an Earth station is located in Singapore (1°N 104'E) then θ_m, the angle between the satellite sub-orbital point and the Earth station, is 73° (more accurately 73°00'08") and using Equation 2.51 the elevation angle of the Earth station antenna can be determined to be 8.4° above the horizon. Then from this elevation angle the propagation path (ℓ) between the satellite and the Singapore station is determined via Equation 2.55 and found to be 25,338 miles.

Example 2.3: Assume an Earth station satellite antenna must be aligned such that for maximum received signal power the antenna is pointed 5° above the horizon.

The distance over which the signal must propagate from the satellite to the Earth station can be determined from Equation 2.53 and is 25,567 miles. If the carrier frequency is known to be 6GHz then the path loss (Equation 2.32) is found to be -200dB.

3. Antenna Alignment Error

As electromagnetic waves propagate from a radiating antenna they expand or "spread" in accordance with the antenna's radiation pattern. From Figure 2.17 , an electromagnetic wave propagated by an antenna with gain g and planar beamwidth ϕ_b, will be spread over a distance W at a distance D units away from the antenna.

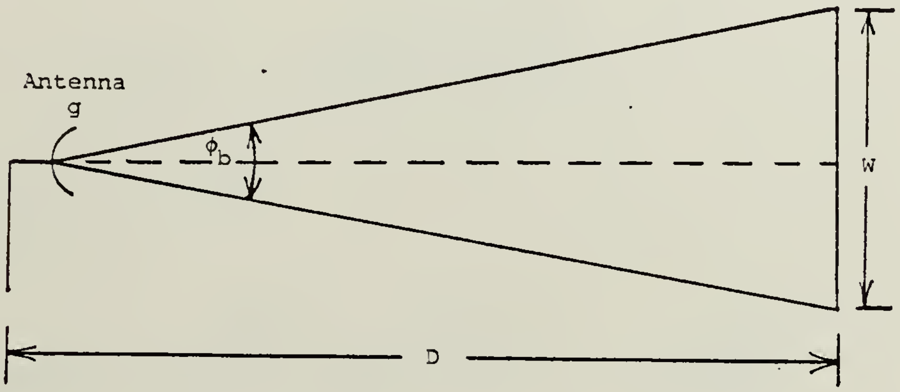


Figure 2.17 Expanding Plane Wave.

The one dimensional spreading distance W of the propagating wave can be determined provided the transmitting antenna gain (g) is known and the spatial radiation pattern is symmetrical. From Equations 2.9 and 2.10 it can be shown that

$$\frac{\phi_b}{2} = 2\arcsin(\frac{1}{g})^{\frac{1}{2}}$$

and from trigonometry:

$$\frac{\frac{1}{2}W}{D} = \tan(\phi_b/2) \;.$$

Then

$$W = 2D\tan(\phi_b/2) \;.$$

Let $\sqrt{(1/g)} = x$ and $\alpha = \arcsin(x)$. Using the trigometric identities $\tan 2\alpha = (2\tan\alpha)/(1-\tan^2\alpha)$, where α is in terms of $\arcsin(x)$, and $\tan[\arcsin(x)] = \sqrt{[x^2/(1-x^2)]}$; then

$$\tan 2\alpha = \frac{2\sqrt{x^2/(1-x^2)}}{1-[x^2/(1-x^2)]} \;.$$

Since $x=\sqrt{(1/g)}$,

$$\tan 2\alpha = \frac{2\sqrt{1/g}\;\sqrt{(g-1)/g}}{(g-2)/g} = \frac{2\sqrt{g-1}}{g-2} \;.$$

Thus

$$W = D\,\frac{4\sqrt{g-1}}{g-2} \;. \qquad \text{(eqn 2.56)}$$

Equation 2.56 indicates that as the gain of an antenna increases the spreading of the plane wave at a given distance decreases. A quantitative comparison of the gains and spreading distances in Table I exemplifies this.

TABLE I

Plane Wave Spread Distance

Antenna Gain (dB)	W/D	W (Miles)
10	1.5000	45.0
20	0.4061	12.2
30	0.1267	3.8
40	0.0400	1.2
45	0.0225	0.7
50	0.0126	0.4

Values are for D=30miles from the transmitter.

In order for an antenna to receive a propagating electromagnetic wave, not only must the receiving antenna be within the area which the propagating wave traverses, but it must also be pointed in the direction of the transmitter. Therefore, if the transmitting and receiving antennas are separated by a distance D, Equation 2.56 (which describes the plane wave spreading distance) determines the distance that a receiving antenna can travel and still be within the field of view of the transmitting antenna. Assuming the antenna in Figure 2.17 describes a receiving antenna, a signal originating at any point along the line W can be "viewed" by the antenna, as long as the signal is propagating toward the receiver. A critical question then follows, "How accurately must antennas be pointed?"

For the case of a geosynchronous satellite which has a transmitting antenna providing Earth coverage (17.4° beamwidth) it can be shown that if the antenna is misaligned 1° from the sub-orbital point all Earth stations on one side of the globe fall outside satellite coverage (see Figure 2.18).

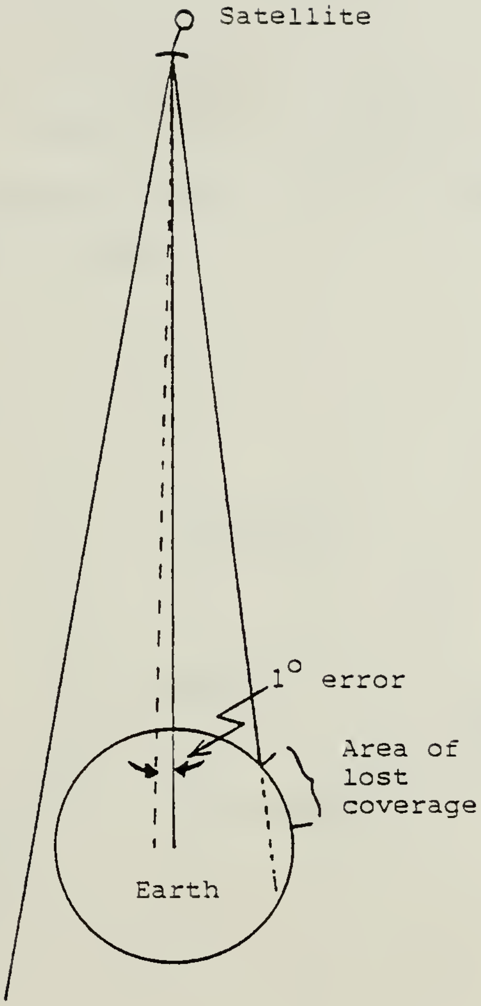


Figure 2.18 1 Degree Satellite Antenna Misalignment.

The misalignment distance S, is given by

$$S = D\theta$$

where D is the distance from the antenna and θ is the misalignment in radians. Thus for a geostationary satellite antenna misalignment of 1°, the transmitted signal is shifted completely out of view for Earth stations within a 388mile fringe where coverage would normally occur.

Conversely, pointing accuracy for an Earth station antenna is not as critical as the pointing of the satellite's antenna. As should be observed, if the Earth station antenna is aligned in such a way that the satellite is in the center of the antenna gain pattern, in order to be within the field of view of the satellite, the pointing accuracy of the antenna must be no worse than half the beamwidth (see Figure 2.19).

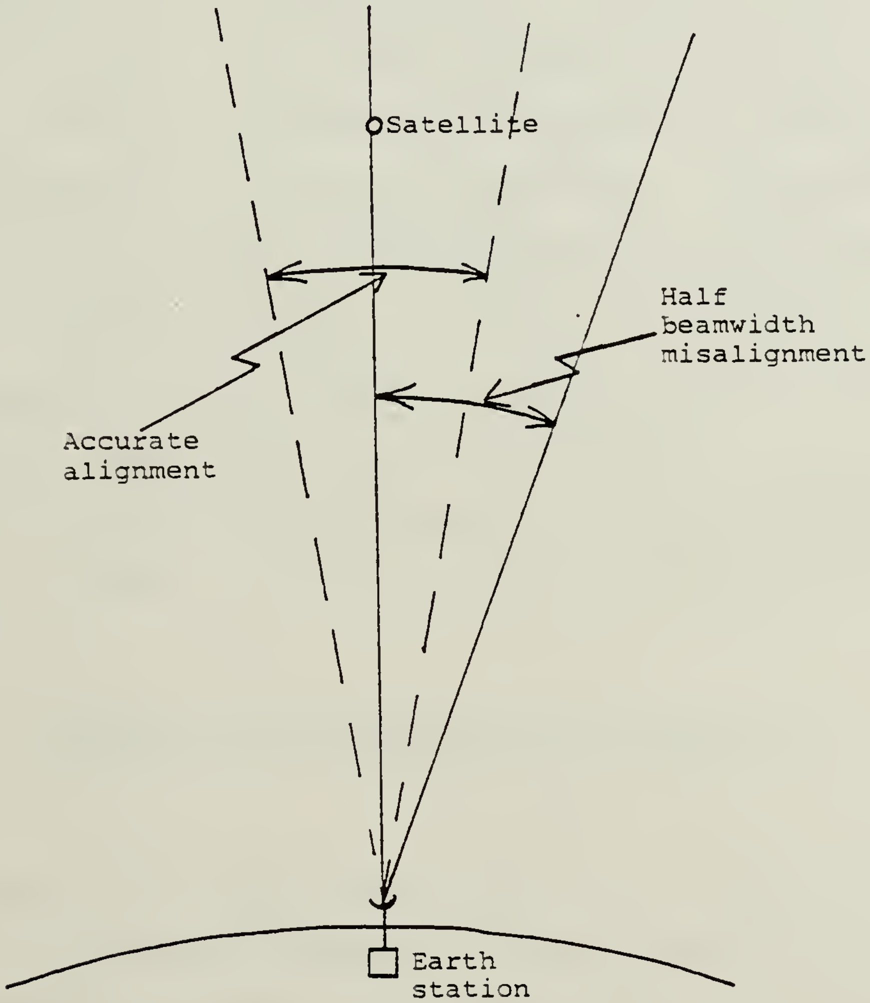


Figure 2.19 Earth Station Antenna Misalignment.

F. PERFORMANCE CRITERION

In the actual operation of a communication system the recovered waveforms at the receiver do not generally correspond exactly to the desired waveforms produced in the transmitter. This is due to anomalies occurring in transmission and reception that cause signal distortion and the insertion of interference and noise waveforms. These effects cause a basic deterioration of the desired waveform and degrade the overall communication operation. To assess the performance of a communication link it is therefore necessary to account for these effects in system analysis. Typically, a specific performance criterion relating desired and actual operation is first decided on. Subsequent system comparison is then based on satisfying the criterion. When more than one system design is being considered, a comparison can be made with respect to the decided criterion and the most favorable system can be determined.

One of the most convenient and widely used measures of performance in communication analysis is the signal to noise ratio (SNR). Signal to Noise Ratio is defined as

$$\mathrm{SNR} = \frac{\text{power in desired waveform}}{\text{power of the interfering waveform}} \,.$$

Thus SNR indicates how much stronger the desired signal is relative to the interference at some point in the system. If SNR is greater than one, there is more power in the desired signal than in the interference, and vice versa if SNR is less than one. **[Ref. 26].**

In addition to collecting the desired carrier field from a transmitting antenna, a receiving antenna also collects noise energy from background sources in its field of view. This background energy is due primarily to random noise

emissions from galactic, solar, and terrestrial sources, constituting the sky background. The amount of noise collected by an antenna is the major limitation to the sensitivity of the receiving system since it determines the weakest carrier signal that can be distinguished. Another type of noise which may appear during reception is radio frequency interference (RFI) and is due to other transmitting sources. A third and final noise to be mentioned is thermal noise. This noise is due to the random motion of electrons within the communication system component material and is directly related to material temperature. As the temperature of the components increases so does electron motion and thus the circuit noise level. Thus thermal noise is zero only at a temperature of absolute zero. [Ref. 27].

The multitude and complexity of factors which affect background noise and RFI preclude further discussion of their contribution to system analysis; however, thermal noise or the noise generated within the communication system warrants further study.

G. SYSTEM NOISE

Consider an electronic device as shown in Figure 2.20 which is being fed by a constant source with output impedance Z_s that is matched to the input resistance Z_{in} of the device. Assume the source is at a temperature T_0 Kelvin and that the power gain of the device is G. Thus it can be seen that

$$P_{out} = P_{in} G \quad .$$

It is well documented that a resistor having impedance Z_s at temperature T_0 generates a thermal noise voltage whose

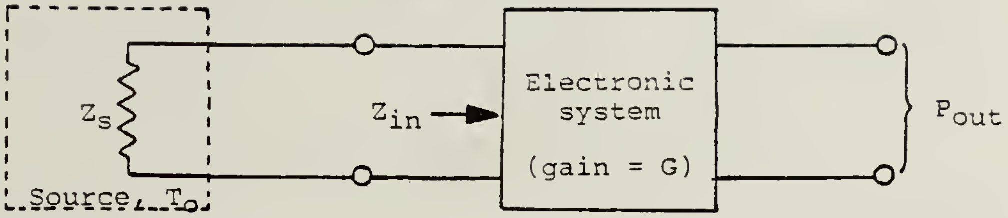


Figure 2.20 Electronic System with Noise Sources.

(voltage) spectral density level is given by $4kT_0 Z_s$ (volts)^2 per hertz, where k is Boltzmann's constant, 1.379×10^{-23} watts/ Kelvin hertz (see Figure 2.21). The noise voltage V generated by Z_s in the bandwidth [0,B] is $4kT_0 Z_s B$. The thermal noise voltage generated by the impedence Z_s can be viewed as an external voltage source so that Figure 2.20 can be viewed in terms of the system shown in Figure 2.22 where Z_s is now considered noise-free. Using the voltage divider principle it can be seen that

$$V = \frac{Z_{in}}{Z_s + Z_{in}} V . \qquad \text{(eqn 2.57)}$$

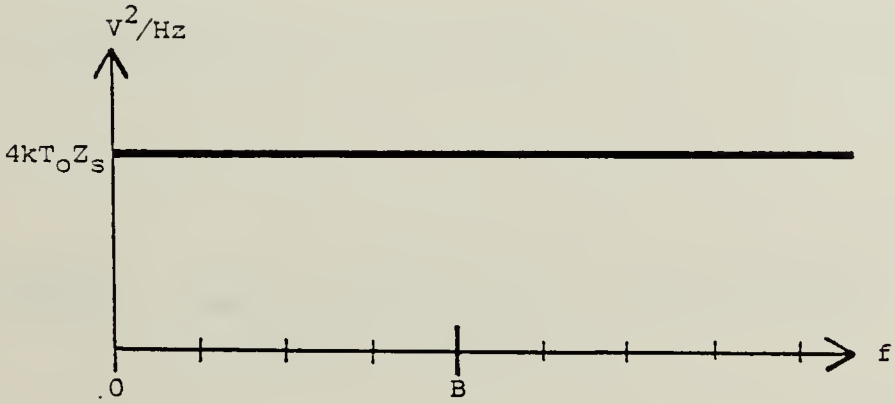


Figure 2.21 Resistor Noise Voltage Spectrum.

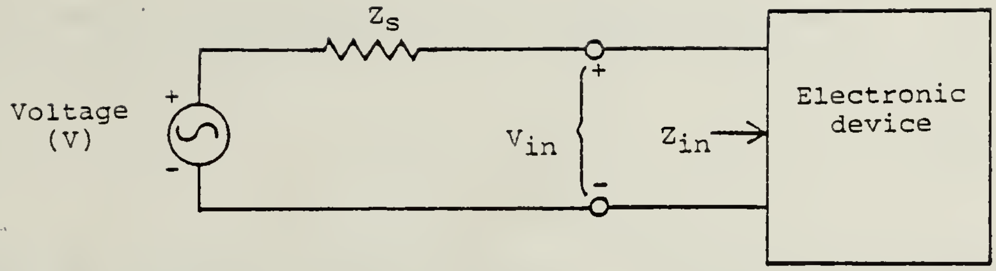


Figure 2.22 Noise Voltage Source to Electronic System.

Since power is given by the voltage squared divided by resistance it can be said that

$$P_{in} = \frac{V_{in}^2}{Z_{in}} \qquad \text{(eqn 2.58)}$$

and substituting Equation 2.57 for V_{in} in Equation 2.58

$$P_{in} = \left[\left(\frac{Z_{in}}{Z_s + Z_{in}}\right)^2 \frac{1}{Z_{in}}\right] V^2 .$$

Using the definition for noise power, P_{in} can be written as

$$P_{in} = \left[\frac{Z_{in}}{Z_s + Z_{in}}\right]^2 \frac{4kT_oZ_sB}{Z_{in}} . \qquad \text{(eqn 2.59)}$$

To achieve maximum power transfer Z_{in} and Z_s must be equal.

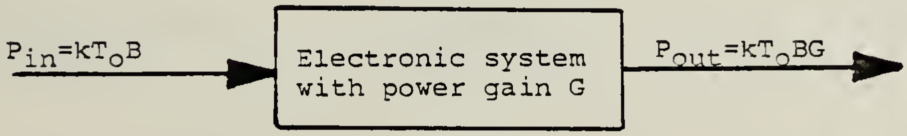


Figure 2.23 System Noise Power.

Setting $Z_{in} = Z_s$ in Equation 2.59 , one obtains

$$P_{in} = kT_0B \qquad \text{(eqn 2.60)}$$

for impedance matched conditions. As shown in Figure 2.23 , the output noise power of a device due to its input thermal noise is given by the product of the device input power and the device power gain. Hence

$$kT_0BG = \begin{array}{l}\text{Noise power output of a device}\\ \text{in B hertz due to the source}\\ \text{thermal noise at } T_0 .\end{array} \qquad \text{(eqn 2.61)}$$

The noise figure F of a device is defined as

$$F \triangleq \frac{\begin{array}{l}\text{Total output noise in a bandwidth B}\\ \text{when input source is at } T_0 = 290^0 \text{ Kelvin}\end{array}}{\begin{array}{l}\text{Output noise due only to the source in}\\ \text{bandwidth B at temperature } T_0 = 290^0 \text{ K}\end{array}} \; . \qquad \text{(eqn 2.62)}$$

Device noise figure therefore is the ratio of the total output device noise to the output noise due to the source alone, when the input source is at 290°K. The total output noise is that noise due to the input source plus that due to internally generated noise within the device itself. Denoting the internally generated noise power as P_{int}, Equation 2.62 becomes

$$F = \frac{k(290^o)BG + P_{int}}{k(290^o)BG} = 1 + \frac{P_{int}}{k(290^o)BG} \; . \qquad \text{(eqn 2.63)}$$

It is readily apparent that $F>1$ for all practical cases since $P_{int} = 0$ only for an ideal noiseless system. Noise figures are usually expressed in decibels with typical values for receiver amplifiers in the range 2-12dB. From

Equation 2.63 the amount of noise generated in a system is given by

$$P_{int} = k[(F-1)290^{0}]BG \ .$$

But T_0 =290° so that

$$P_{int} = k[(F-1)T_0]BG \ . \qquad \text{(eqn 2.64)}$$

Comparing Equations 2.60 and 2.64 it can be seen that the internal noise can be viewed as if it were caused by an input system temperature $(F-1)T_0$. This temperature, $(F-1)T_0$, is called the device equivalent temperature T so that

$$T_{eq} = (F-1)T_0 \ . \qquad \text{(eqn 2.65)}$$

Example 2.4: Assume a device has a noise figure of 3dB. Then F=2 and the equivalent temperature of the device is given by

$$T_{eq} = (2-1)\,290^{o}K$$

$$T_{eq} = 290^{o}K \ .$$

Example 2.5: Assume a device operates at an equivalent temperature T_{eq}=70°K, then the device noise figure can be determined by:

$$70^{o} = (F-1)\,290^{o}.$$

Thus the noise figure (F) = (70/290)+1 = 1.24 or 0.94dB.

In most cases an electronic device is composed of a series or cascade of devices each having its own noise figure and power gain. In order to determine the total noise power of the system, the noise figure of each stage of the cascade must be evaluated. For analysis purposes consider the cascade in Figure 2.24 ; each device has noise figure (NF) F_1 and F_2 and power gain G_1 and G_2 respectively. Assume the devices are impedance matched at their respective input and output terminals. Then given that $P_{in1}=kTB$ and from Equations 2.62 and 2.64

$$P_{out1} = kT_0 BG + P_{int1}$$

where

$$P_{int1} = k[(F_1 - 1)T_0]BG.$$

Therefore

$$P_{out1} = kT_0 BG_1 + k[(F_1 - 1)T_0]BG_1$$

$$P_{out1} = kT_0 BG_1 [1 + (F_1 - 1)]$$

$$P_{out1} = kT_0 BG_1 F_1 .$$

Now from Figure 2.24 and assuming impedance matched conditions, it can be seen that $P_{out1} = P_{in2}$ so that

$$P_{out2} = P_{in2} G_2 + P_{int2}$$

$$P_{out2} = P_{out1} G_2 + k[(F_2 - 1)T_0]BG_2$$

$$P_{out2} = kT_0 BG_1 F_1 G_2 + k[(F_2 - 1)T_0]BG_2 .$$

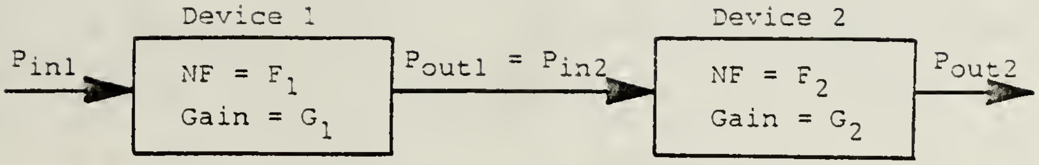


Figure 2.24 Cascade of Two Systems.

Solving for the total noise figure according to Equation 2.62

$$F = \frac{kT_0BG_1G_2F_1 + k(F_2-1)T_0BG_2}{kT_0BG_1G_2}$$

$$F = F_1 + \frac{F_2-1}{G_1} \quad . \qquad \text{(eqn. 2.66)}$$

Following a similar approach, an analysis of a system containing several cascaded devices can be conducted. In general a cascade of n devices results in an overall system noise figure equation of

$$F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2} + \frac{F_4-1}{G_1G_2G_3} + \cdots\cdots + \frac{F_n-1}{G_1G_2G_3\cdots G_{n-1}} \qquad \text{(eqn 2.67)}$$

where F_i represents the noise figure of the i^{th} stage; G_i represents the gain of the i^{th} stage; and i=1,2,3,...n. It should be noted that in Equation 2.67 F_i and G_i are numbers (that is, not given in dB's).

Example 2.6: Assume a system is composed of three noisy devices in cascade each with the noise figures and gains indicated in Figure 2.25 . The overall system noise figure can then be determined using Equation 2.67 so that

$$F = 2 + \frac{6.31 - 1}{100} + \frac{3.16 - 1}{(100)(1.023)}$$

$$F = 2 + 0.0531 + 0.0211$$

$$F = 2.0742$$

Comparing the resultant noise figure to the noise figure of the first stage alone, it can be observed that had the overall noise figure been estimated as equal to the first section noise figure only, a small error would result.

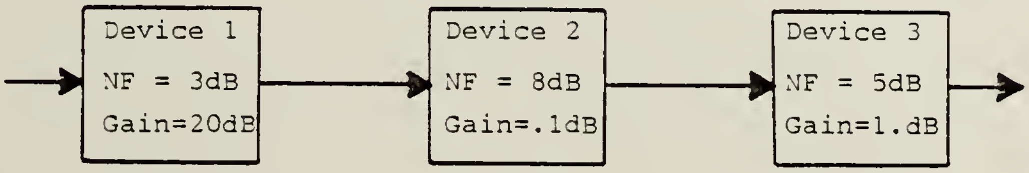


Figure 2.25 Cascade of Three Noisy Devices.

Examination of Equation 2.67 indicates that each successive stage of a cascade device contributes less and less to the overall system noise figure. Furthermore, the first stage contribution is its entire noise figure while the contribution of the second is some inverse proportion of the gain of the first device. Therefore, it can be concluded that when considering system noise figure, the most important factors for a cascaded system is the first stage noise figure and gain characteristics.

Because thermal noise is caused by the random motion of electrons and systems cannot be operated to absolute zero temperature, every electronic device must necessarily have a noise figure associated with it that is greater than unity. Consider a purely lossy or attenuation device such as a transmission line, waveguide or cable at a temperature (T_0) (Figure 2.26). Let L_g be the power loss factor of the attenuating device. Assuming a matched input power source is at temperature T_g, then the input noise power to the attenuator is given by Equation 2.60 so that

$$P_{in} = kT_gB \quad . \qquad \text{(eqn 2.68)}$$

Then from Equation 2.61 the noise power out of the attenuator is

$$P_{out} = kT_gBL_g \qquad \text{(eqn 2.69)}$$

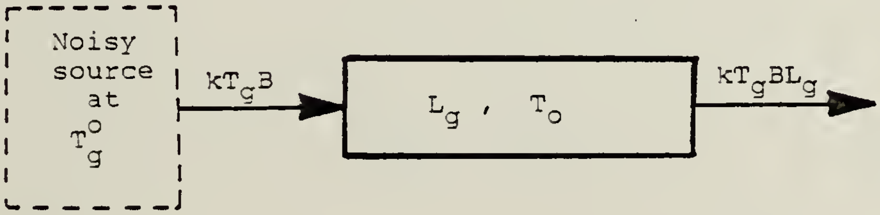


Figure 2.26 Noisy Attenuator.

and since $L_g < 1$, then $P_{out} < P_{in}$ so that the difference in the power levels must be the noise power dissipated by the attenuator and therefore is the internal noise power generated by the device. It can then be seen that for any lossy

device

$$P_{in} = P_{out} + P_{int} \qquad \text{(eqn 2.70)}$$

so that the internal noise power can be determined from

$$P_{int} = P_{in} - P_{out} .$$

Substitution of Equations 2.68 and 2.69 into Equation 2.70 results in

$$P_{int} = kT_gB - kT_gBL_g . \qquad \text{(eqn 2.71)}$$

Now to determine the noise figure of the lossy device Equation 2.71 can be substituted into Equation 2.63 thus

$$F = 1 + \frac{kT_gB(1 - L_g)}{kT_0BL_g}$$

or

$$F = 1 + \frac{1}{L_g} - 1 \frac{T_g}{T_o} . \qquad \text{(eqn 2.72)}$$

It can be concluded that when the input source to a lossy device is at T_0^o, then the noise figure of that lossy device is given as

$$F = \frac{1}{L_g} . \qquad \text{(eqn 2.73)}$$

Attenuating devices such as waveguides, cables and transmission lines are grossly specified in terms of their power loss per unit length, such as for example 0.1dB per meter. Thus an attenuating device's noise figure changes as its length is altered.

Example 2.7: Determine the noise figure of a 50meter long coaxial cable having a 0.1dB/meter loss at a temperature of 350°K.

$$(L_g)\,dB = (50\text{meters}) X (0.1dB/\text{meter}) = 5dB.$$

Thus the total cable loss of 5dB translates to L_g=0.316 so

$$F = 1 + [(1/L_g)-1]T_g/T_0$$

$$F = 1 + [(1/.316) - 1](350/290)$$

$$F = 3.61 \text{ or } 5.5 \text{ dB}.$$

What is the noise figure if 10meters are cut from the cable? It is readily apparent that the loss of the cable is now 4dB which represents an L_g of 0.398 and therefore

$$F = 1 + [(1/.398) - 1](350/290)$$

$$F = 2.2$$

or equivalently

$$F = 3.4 \text{ dB}.$$

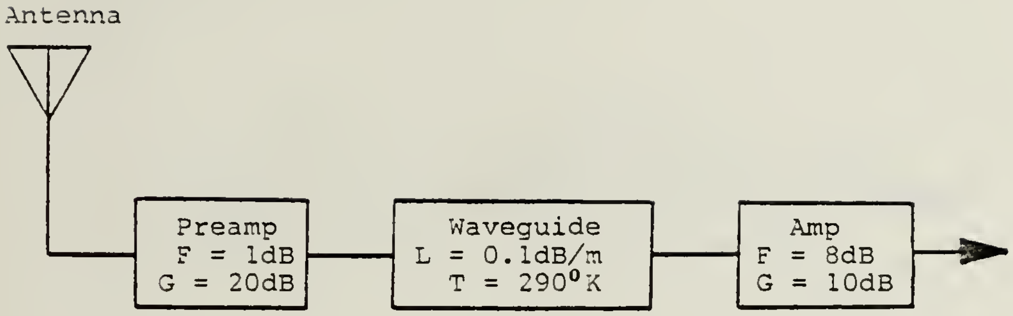


Figure 2.27 Receiver System Front-End Block Diagram.

By applying the underlying concepts of Equations 2.67 and 2.73 , the overall noise figure of a system composed of a cascade of amplifiers and attenuators can be obtained.

Example 2.8: Given a receiver front-end block diagram in Figure 2.27 for which the maximum acceptable system noise figure is 10dB, the maximum allowable length of the waveguide can be determined.

$$F_{Total} = F_{preamp} + \frac{F_{guide} - 1}{G_{preamp}} + \frac{F_{amp} - 1}{G_{preamp} L_{guide}}$$

$$F = 1.26 + \frac{1/L_g - 1}{100} + \frac{6.32 - 1}{100 L_g} \quad .$$

Now F_{Total} = 10dB, so

$$10 = (1.26) + (1/100L_g) - (1/100) + (5.3/100L_g)$$

$$8.75 = (6.3/100L_g)$$

$$L_g = 6.3/875 = .00721.$$

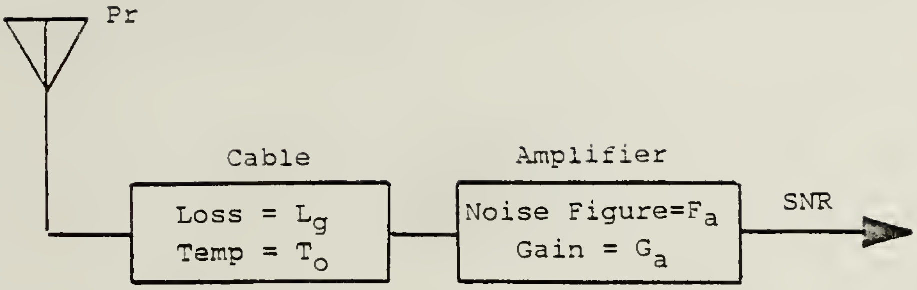


Figure 2.28 Receiver Front-End.

Thus

$$L_g = -21.42 \text{ dB}$$

so that

$$\text{length of guide} = \frac{-21.42 \text{ dB}}{-0.1 \text{ dB/meter}} = 214.2 \text{ meters.}$$

Therefore, the maximum length the waveguide can be such that the overall system noise figure will not exceed 10dB is 214.2meters.

As mentioned in the previous section, one of the performance criteria used to evaluate performance of electronic systems is their output signal to niose ratio (SNR). The SNR compares the desired information signal power to the internal noise power level. A simple expression for SNR is developed through use of an example, as illustrated in Figure 2.28 . Assume the received carrier power is P_r , the cable connecting the antenna to the amplifier has a loss factor L_g and is at T_0 Kelvin. The amplifier has a noise figure F_a and a gain of G_a. The overall system noise figure

F is determined to be (F_a/L_g) from Equations 2.67 and 2.73. Equation 2.63 indicated that

$$F = 1 + \frac{P_{int}}{kT_0 B L_g G_a} \ .$$

It can be seen that the output noise power (P_{int}) from the system is given by

$$P_{int} = [(F_a/L_g) - 1] kT_0 B L_g G_a \ .$$

If the signal or carrier input power to the system is P_r, the output signal power from the system is given by $P_r L_g G_a$. Thus

$$SNR = \frac{P_r L_g G_a}{[(F_a/L_g) - 1] kT_0 B L_g G_a} \ .$$

or

$$SNR = \frac{P_r}{[(F_a/L_g) - 1] kT_0 B} \ .$$

Recognizing that F_a/L_g is the overall system noise figure in the example, a generalized system SNR can now be stated as

$$SNR = \frac{P_r}{(F - 1) kT_0 B} \ . \qquad \text{(eqn 2.74)}$$

B. MICROWAVE RELAY SYSTEM ANALYSIS

In 1947, the Bell Telephone System placed the first microwave carrier system in operation. Microwave systems utilize line-of-sight transmission; therefore, the system transmitters and receivers must be situated such that they can "see" each other along the curvature of the earth. This is accomplished by mounting the systems' antennas on towers high above ground. For economic reasons, towers of approximately 100feet are commonly used. Thus the maximum distance between each tower can be determined by Equation 2.38 . The analysis of a microwave system is initially approached via the use of the power budget equation. A more practical approach that accounts for construction limitations is then analyzed.

1. Theoretical Approach

Assume a signal must be transmitted over some distance D_T which is beyond line-of-sight. In this case an intermediary relay must be installed as shown in Figure 2.29. Also assume that the relays are equally spaced at some ΔD interval and each has a power amplifier with a gain of A. That is for any given relay, $P_{out} = P_{in} A$, or equivalently

$$(P_{out})\,dB = (P_{in})\,dB + (A)\,dB.$$

Disregarding noise, assume each transmit and receive antenna has a gain of G_t and G_r respectively. Also assume all losses except propagation path loss are combined in one term labled L. Then refering to Figure 2.29

$$(P_{t1})_{dB} = (P_{in})_{dB} + (A)_{dB}$$

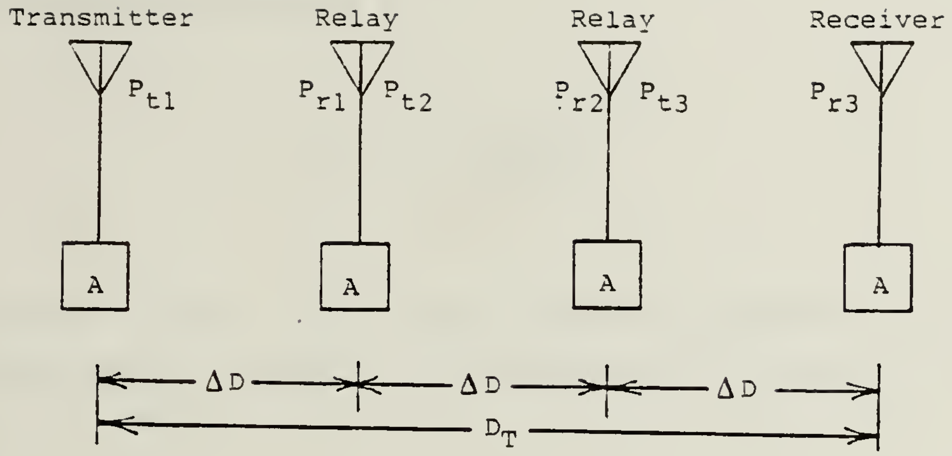


Figure 2.29 Microwave Relay System.

$$(P_{r1})_{dB} = (P_{in})_{dB} + (A)_{dB} + (L)_{dB} + (L_p)_{dB} + (G_t)_{dB} + (G_r)_{dB}$$

$$(P_{r2})_{dB} = (P_{r1})_{dB} + (A)_{dB} + (I)_{dB} + (L_p)_{dB} + (G_t)_{dB} + (G_r)_{dB}$$

$$(P_{r3})_{dB} = (P_{r2})_{dB} + (A)_{dB} + (I)_{dB} + (L_p)_{dB} + (G_t)_{dB} + (G_r)_{dB}$$

Substituting P_{ri} into successive equations it can be seen that

$$(P_{r3})_{dB} = (P_{in})_{dB} + 3[(A)_{dB} + (I)_{dB} + (L_g)_{dB} + (G_t)_{dB} + (G_r)_{dB}].$$

Again referring to Figure 2.29 it can be demonstrated that if a communication link over a distanct D_T is broken into n equal sections of distance ΔD then the number of repeaters which will be installed is $n-1$. Therefore the power received by the n^{th} receiving antenna is given by

$$(P_{rn})_{dB} = (P_{in})_{dB} + n[(A)_{dB} + (L)_{dB} + (L_g)_{dB} + (G_t)_{dB} + (G_r)_{dB}].$$

L_p was previously defined by

$$L_p \triangleq \left[\frac{c}{4\pi Df}\right]^2$$

and in Figure 2.29 D is the distance related to the path loss between each relay. Therefore for each L_p factor in the equation for P_{rn}

$$L_p = \left[\frac{c}{4\pi \Delta Df}\right]^2 .$$

Since $D_T/n = \Delta D$, then for this analysis of the system depicted in Figure 2.29,

$$L_p = \left[\frac{nc}{4\pi D_T f}\right]^2 .$$

Therefore any microwave relay system can be analyzed in terms of the power received at the receiver site antenna where the received power is given by

$$(P_{rn})_{dB} = n\left[A + L + 20\log\frac{nc}{4\pi f D_T} + G_t + G_r\right] + (P_{in})_{dB} \qquad \text{(eqn 2.75)}$$

and all factors are dB values.

Example 2.9: The one link system. Suppose that L=3dB; A=20dB; P_{in}=1watt; $G_t + G_r$ =40dB ; distance =100Km; and f=10GHz; the power received can be determined from Equation 2.75 where n=1 for this case. First, however, (L_p) dB is determined from Equation 2.36 so that

$$L_p = (-32.44) - 20(\log 100) - 20(\log 10) = -152.44 \text{ dB}.$$

Then from Equation 2.75

$$P_{r1} = (20dB - 3dB - 152.44dB + 40dB) + 0dBW$$

$$P_{r1} = -95.44 \text{ dBW}.$$

Example 2.10: The two link system. Given the same values of the previous example with the exception that a two link system over a 100 km distance results in a distance of 50Km between any transmit-receive antenna pair. The propagation path loss factor (L_p) is now reduced to

$$L_p = -32.44 - 20\log(50) - 20\log(10) = -146.42dB.$$

Then

$$P_{r1} = 20dB - 3dB - 146.42dB + 40dB = -89.42dBW$$

and

$$P_{r2} = (20dB - 3dB - 146.42dB + 40 \text{ dB}) - 89.42dBW$$

$$(P_{r2}) \text{ dB} = -178.84 \text{ dBW}.$$

If a system is to be installed over a given distance and a minimum level of received power is necessary and known, the minimum number of relays needed can be determined using Equation 2.75 .

Example 2.11: Assume $P_{rn} \geq -15$dB; $P_{in} = 1$watt; $A = 30$dB; $I = 3$dB; $f = 1$GHz; $G_t + G_r = 90$dB; $D = 100$Km. The minimum number of relays needed is determined from

$$P_{rn} = n[30 - 3 + 20\log(nc/4\pi fD) + 90] + 0$$

$$-15\text{dB} = n[-15.44 + 20\log(n)]\text{dB}.$$

Solution of this nonlinear equation in n is best accomplished by trial and error method, so that for this example n can be determined from Table II. Clearly Table II indicates that the minimum number of microwave links is 4. Therefore, two relay towers should be installed between the transmitter and receiver in this microwave system.

TABLE II

Solution to Example 2.11

n	D (Km)	P (dB)
1	100.0	-15.44
2	50.0	-18.84
3	33.3	-17.70
4	25.0	-13.60

2. Applications Approach

A more realistic problem is one in which the total transmission distance is known and a specified number of links is required. The problem is to determine the power level requirements at the receiver.

Example 2.12: Assume that P_{in} is 1watt; A=30dB; L=3dB; G_t+G_r =90dB; f=1GHZ; distance of entire system is 500Km; and 10 links are to be used. The minimum received power level can be determined to be

$$P_{rn} = n[A+L+L_g+G_t+G_r]+P_{in}.$$

First

$$L_p = -32.44dB - 20\log(500/10) - 20\log(10) = 126.42dB.$$

Then

$$P_{rn} = 10[30dB - 3dB - 126.42dB + 90dB] + 0dBW$$

$$P_{rn} = -94.2dBW.$$

Normally, technical considerations dictate the minimum discernable signal level required at the input of a receiver. Thus Equation 2.75 can be used to determine the required input signal power. In addition, the Federal Communication Commission (FCC) or other regulatory agencies may limit transmitter power and operating frequencies; therefore Equation 2.75 can be used to determine total antenna gain parameters. Assuming the carrier frequency and the number of relays are determined prior to system design, with regard to the variables in Equation 2.75, only the loss factors can be considered physical properties which the system designer can not alter.

3. Microwave System Signal-to-Noise Ratio

As mentioned in earlier segments of this chapter, system signal-to-noise ratio is the predominant parameter by which a system's performance is evaluated. It is necessary then that some effort be made to develop an understandable approach to the determination of SNR for the microwave relay system.

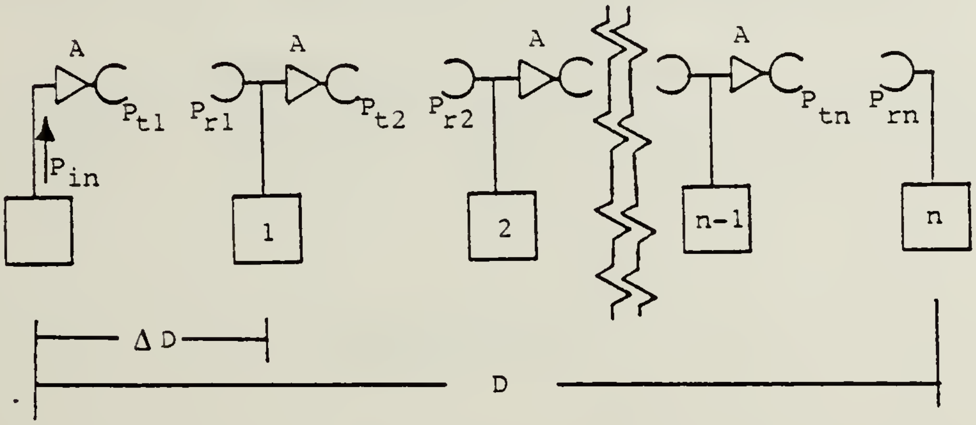


Figure 2.30 Multi-link Microwave System.

Assume a microwave relay system consists of n links of equal distance as illustrated in Figure 2.30 . Further assume that similar components are used for each link such that the losses from propagation, atmospheric conditions (i.e. rain or suspended particulate matter), and coupling can be "lumped" together into an aggregate gain term G. Also, if the noise figure and gain of the amplifier are known and denoted F_a and A respectively, then Figure 2.30 can be viewed as being represented by Figure 2.31 . Let the output power of link 1 be denoted $P_{out\,1}$ such that

$$P_{out\,1} = P_{s1} + P_{n1}$$

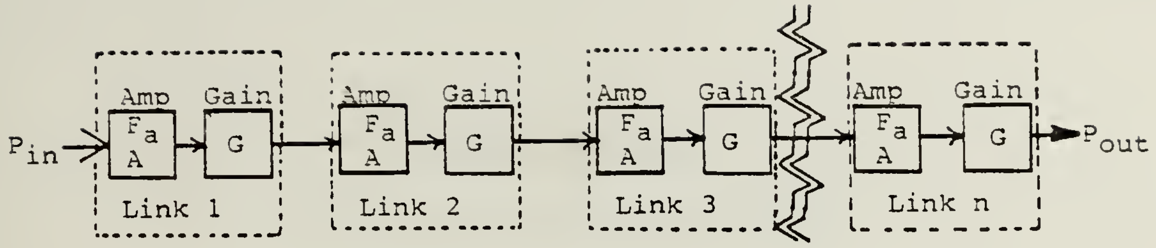


Figure 2.31 Simplified Multi-link System.

where P_{s1} is the signal power out of Link 1 and P_{n1} is the noise power out. Then the signal power out of Link 1 is given by:

$$P_{s1} = P_{in} AG$$

where P_{in} is the signal power input to Link 1 and the noise power output of Link 1 is

$$P_{n1} = P_{int} G$$

where P_{int} is the noise power generated by the amplifier. From Equation 2.64 it has been determined that

$$P_{int} = (F_a - 1) kT_o BA$$

so that

$$P_{n1} = (F_a - 1) kT_o BAG.$$

Therefore the signal-to-noise ratio for the first link of the system is

$$SNR_1 = P_{s1}/P_{n1}$$

$$SNR_1 = P_{in}AG/(F_a - 1)kT_0 BAG$$

$$SNR_1 = \frac{P_{in}}{(F_a - 1)kT_0 B} \quad . \qquad \text{(eqn 2.76)}$$

It can be readily seen that in Figure 2.31 the output of Link 1 is the input to Link 2 and consists of both signal (P_{s1}) and noise (P_{n1}) . The output of Link 2 will be an amplification of both signal and noise plus some additional noise generated in the link 2 amplifier. The signal power out of Link 2 is

$$P_{s2} = P_{s1}AG$$

but

$$P_{s1} = P_{in}AG$$

so

$$P_{s2} = P_{in}(AG)^2 \; .$$

The noise power out of Link 2 is

$$P_{n2} = (P_{n1}AG) + (P_{int}G)$$

but

$$P_{int} = (F_a - 1)\, kT_o BA$$

and

$$P_{n1} = (F_a - 1)\, kT_o BAG.$$

Thus

$$P_{n2} = (F_a - 1)\, kT_o B\, (AG)^2 + (F_a - 1)\, kT_o BAG$$

or

$$P_{n2} = (F_a - 1)\, kT_o B[\, (AG)^2 + (AG)\,]\ .$$

Therefore the signal-to-noise ratio at the output side of Link 2 is given by

$$SNR_2 = P_{s2}/P_{n2}$$

$$SNR_2 = P_{in}\, (AG)^2/(F_a - 1)\, kT_o B\ [\, (AG)^2 + (AG)\,]\ .$$

It can be shown then that by applying Equation 2.76 the system SNR at the end of two links is given by

$$SNR_2 = SNR_1 \left\{ 1/[\, 1 + (AG)^{1}\,] \right\}$$

where SNR_1 is the signal-to-noise ratio of the first link. Continuing in like manner

$$P_{s3}=P_{s2}(AG)=P_{s1}(AG)^2=P_{in}(AG)^3$$

and

$$P_{n3}=P_{n2}AG+P_{int}=P_{n1}(AG)^2+P_{int}(AG)^2+P_{int}G$$

or

$$P_{n3}=(F_a-1)kT_0B(AG)^3+(F_a-1)kT_0B(AG)^2+(F_a-1)kT_0B(AG).$$

Thus

$$SNR_3=P_{s3}/P_{n3}$$

$$SNR_3=P_{in}(AG)^3/(F_a-1)kT_0B[(AG)^3+(AG)^2+(AG)]$$

which simplifies to

$$SNR_3=SNR_1\left\{1/[1+(AG)^1+(AG)^2]\right\}$$

where SNR_1 is the signal-to-noise ratio of the first relay link. To determine the signal-to-noise ratio of an n link microwave relay system, the following equation applies

$$SNR=SNR\,\frac{1}{\sum_{k=1}^{M}(AG)^{1-k}}\,. \qquad \text{(eqn 2.77)}$$

since

$$\sum_{k=1}^{M}\left(\frac{1}{AG}\right)^{k-1} = \sum_{k=0}^{M-1}\left(\frac{1}{AG}\right)^{k},$$

from the well-known closed form sum for a geometric series, namely:

$$\sum_{k=0}^{M-1} x^k = \begin{cases} \dfrac{1 - x^M}{1 - x} & \text{for } x \neq 1 \\ M & \text{for } x = 1 \end{cases}$$

Equation 2.77 can be expressed accordingly so that the overall SNR at any given receiver within the system can be determined from

$$SNR_n = \begin{cases} SNR_1 \left[\dfrac{1 - \left(\frac{1}{AG}\right)}{1 - \left(\frac{1}{AG}\right)^M}\right] & \text{for } AG \neq 1 \\ SNR_1 \left[\dfrac{1}{M}\right] & \text{for } AG = 1 \end{cases}$$

and

$$SNR_1 = P_{in}/[(F_a - 1)\, kT_0\, B]$$

where F_a is the noise figure and A is the gain of the amplifier used in the link, P_{in} is the signal input power to the transmitter, and G is the aggregate gains and losses from the transmitter amplifier output to the receiver portion of a single link.

Example 2.13: Assume a 400Km microwave system consists of seven relay towers equally spaced between a transmitting and receiving station. The power amplifiers in the system each have a 60dB noise figure and a gain of 60dB within a 10MHz bandwidth. The signal power into the system is 1watt and each antenna has 25dB gain. Assume coupling and atmospheric attenuation account for a combined loss of 3 dB per link. The SNR after the first link is easily determined. If the carrier frequency is centered at 1000MHz then using the propagation loss equation (2.35), the propagation loss per link is found to be -130.6dB. Thus the aggregate loss between the output side of any of the power amps and the front-end of the subsequent receiver is -54dB. Thus

$$SNR_1 = 1/[(10^6 - 1)(1.379\times10^{-23})(290)(10^7)] = 2.5\times10^7$$

$$SNR_1 = 74\text{dB}.$$

Recall that for the SNR equation all dB values must be changed to their respective ratio values. Then the SNR for the system at the second link can be determined from

$$SNR_2 = 2.5\times10^7 \left\{1/[1+(AG)^{-1}]\right\}.$$

The gain of all amplifiers is 60dB so that $A=10^6$ and the aggregate gain (loss) of the transmission channel is -54dB or $G=4.4\times10^{-6}$. Thus the total value for (AG) is 4.4. Therefore,

$$SNR_2 = 2.5\times10^7 (1/1.23) = 2\times10^7$$

so that the SNR after the second link is 73dB. Finally at the end of the eighth link the system SNR is found by use of the equivalent closed form sum value so that

$$SNR_8 = 2.5x10^7 \left[\frac{1 = \left(\frac{1}{AG}\right)}{1 = \left(\frac{1}{AG}\right)^8} \right]$$

$$SNR_8 = 2.5x10^7 \left[\frac{1 - 0.23}{1 - 0.000007} \right]$$

$$SNR_8 = 1.9x10^7$$

Thus the overall system SNR at the end of the total link is 72.8dB.

As can be easily observed from both Equation 2.74 and the previous example, the microwave relay system signal-to-noise ratio is dominated by the signal-to-noise ratio of the first link. Therefore, careful consideration of the transmitter's power amplifier noise characteristics must be made when designing or specifying contractually a microwave system.

III. TRANSMISSION LINES

It should be of no surprise that the transmission line was the first communication channel used for electronic communications [Ref. 28]. Transmission lines formed the infrastructure of the American Telephone and Telegraph system and until the latter part of this century were the only means of establishing transoceanic communication networks. Transmission lines are generally grouped into several classes according to their cross-sectional geometry. The principal classes are: (1) balanced open-wire lines; (2) coaxial lines; and (3) strip lines. However, all of these consist of a pair of parallel wires and therefore have certain properties in common which allows a generalized analysis.

An essential characteristic of all transmission lines is that their cross-sectional shape, dimensions, and electrical properties (e.g., conductivity of the wires and dielectric constant of the interconducter medium) are constant along the length of the line. This property, known as uniformity, is characteristic of transmission lines in general and is the basis for subsequent transmission line analysis results. Furthermore, a transmission line is considered as passive and therefore is a "lossy" or attenuating device. Because of line uniformity, transmission lines are grossly specified in terms of their power loss per unit length. This power loss or attenuation per unit length is denoted by α and normally stated in dB's. Given the general communication system illustrated in Figure 3.1 the total transmission line attenuation (L) in decibels is determined by multiplying the line attenuation factor (α) by the total length of the line

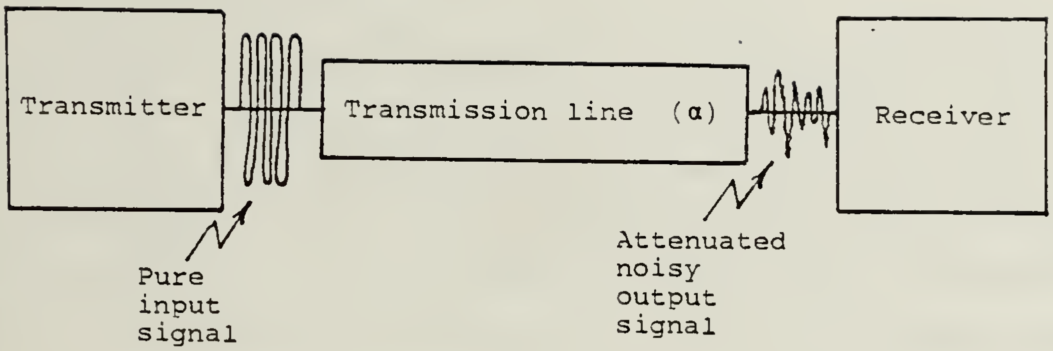


Figure 3.1 **Transmission Line Communication Channel.**

(D), so that

$$L_{dB} = \alpha D \; . \qquad \text{(eqn 3.1)}$$

Unlike the study of antenna systems which focuses on the decreasing field density of an expanding electromagnetic wave which must therefore consider external signals or noise sources as potential degradation factors affecting system performance, analysis of the transmission line can generally ignore outside signal sources since the communication signal is confined to the transmission line as well as protected from general interference from external signals. The signal-to-noise ratio criteria used to evaluate system performance can then ignore all but thermal noise generated by the random molecular motion within the transmission line component materials as discussed in Chapter 2.

Contrasted with the previous chapter which first developed a detailed study of antenna system parameters and culminated in a broad analysis of a microwave relay system, this chapter begins with a generalized analysis of various transmission line configurations followed by the details of the line characteristics.

A. GENERAL SYSTEM ANALYSIS

The simplest transmission line system is one composed solely of a two wire conductor placed between the transmitter and receiver sections of a communication system as in Figure 3.1 Assuming the unit length attenuation factor (α) of the transmission line is known, the total attenuation of the line is determined by Equation 3.1 It is known that the ratio of system power output to system power input describes the system's overall gain or loss. Using Figure 3.2 it can be seen that

$$\frac{P_{out}}{P_{in}} = \text{Loss} \qquad \text{(eqn 3.2)}$$

where the loss (L) in dB's is given by $L_{dB} = \alpha D$. But the power out-to-power in ratio is not a dB figure; thus L must be converted from dB notation such that:

$$L = 10^{-0.1 \alpha D} . \qquad \text{(eqn 3.3)}$$

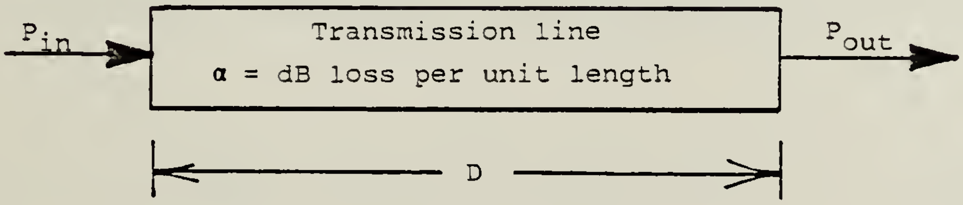


Figure 3.2 Simple Transmission Line System.

It should be noted that L represents a loss and must therefore be less than 1. For this to hold in Equation 3.3 it must be realized that the α term is negative.

As discussed in Section F of Chapter 2, a system's signal-to-noise ratio (SNR) is an important criterion in the evaluation of a system's performance. Since thermal or internal noise is considered the only source of noise which affects signals on a transmission line, the system SNR is given by

$$\text{SNR} = \frac{\text{power of signal out}}{\text{power of internal noise}} \quad .$$

From Equation 3.2 it can be determined that

$$P_{out} = P_{in} L \qquad \text{(eqn 3.4)}$$

and from Section G of Chapter 2 the noise power or internal noise of a lossy device is given by

$$P_{int} = \left(\frac{1}{L} - 1\right) kT_0 BL \quad . \qquad \text{(eqn 3.5)}$$

Therefore, by substituting Equations 3.4 and 3.5 into the equation for SNR and cancelling the loss factor which appears in both the numerator and denominator it can be seen that:

$$\text{SNR} = \frac{P}{\left(\frac{1}{L} - 1\right) kT_0 B} \quad . \qquad \text{(eqn 3.6)}$$

Example 3.1: Assume a simple transmission system consists of a 10km cable which has a loss factor of 1 dB per km. If a 1microwatt information signal with a 100KHz bandwidth is placed on the transmission line, the system SNR can

be determined by applying Equation 3.6 resulting in

$$\mathrm{SNR} = \frac{10^{-6}}{(10 - 1)(1.379\times10^{-23})(290)(10^{5})} = 2.778\times10^{8} .$$

Since SNR's are usually specified in decibels, for this example SNR = 84.4 dB.

With the exception of short distance networks such as inter-office or intra-building systems, transmission line systems consisting solely of a two wire line exhibit severe signal attenuation. If severe attenuation occurs, satisfactory system operation requires the use of extremely high transmitter power and/or extremely sensitive receivers. An alternate approach to the problem of high attenuation is to insert an amplifier some distance along the line as illustrated in Figure 3.3 . To evaluate system performance, the overall SNR must be determined. If noise effects are not considered, then the placement of the amplifier is of no consequence. Assume the transmission line in Figure 3.3 has an attenuation factor α, distances D_1 and D_2 equal the total transmission line system distance D, and the amplifier has gain A. Letting L_1 and L_2 represent the loss of the lines of length D_1 and D_2 respectively, Equation 3.3 indicates

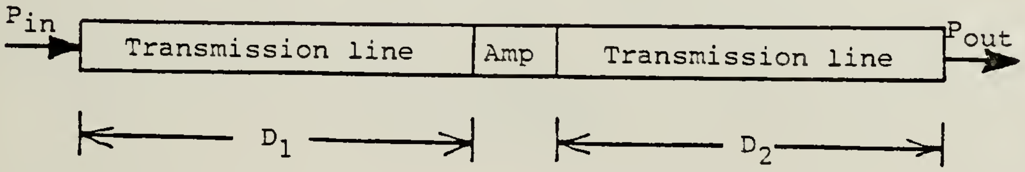


Figure 3.3 Cable Model with Single Amplifier.

that

$$L_1 = 10^{-0.1\alpha D}$$

and

$$L_2 = 10^{-0.1\alpha D} .$$

From the definition of gain/loss it can be shown that

$$L_1 = \frac{P_{out1}}{P_{in1}}$$

where $P_{in\,1}$ and $P_{out\,1}$ are the input and output power levels, respectively, of the first line segment and

$$L_2 = \frac{P_{out2}}{P_{in2}}$$

where $P_{in\,2}$ and $P_{out\,2}$ are the input and output power levels, respectively, of the second segment of line. Referring to Figure 3.3 it can be seen that the output of the first transmission line is the input to the amplifier. Designating the amplifier's input and output powers P_1 and P_2 respectively, it should be realized that

$$P_1 = P_{in\,1} \; L_1$$

and

$$P_2 = P_i A = P_{in} L_1 A .$$

Since the input to the second line is also the output of the amplifier, the output of the second portion of the transmission line is

$$P_{out2} = P_2 \; L_2 = P_{in} \; L_1 A \; L_2$$

so that the system output power is

$$P_{out} = P_{in} A \; L_1 L_2 \; .$$

Substituting for L_1 and L_2

$$P_{out} = P_{in} A \; 10^{-0.1 \alpha D_1} 10^{-0.1 \alpha D_2}$$

or

$$P_{out} = P_{in} \; A \; 10^{-0.1 \alpha (D_1 + D_2)} \; .$$

Now since $D_1 + D_2 = D$ (the total transmission line distance) it can be seen that

$$P_{out} = P_{in} \; A \; 10^{-0.1 \alpha D}$$

where $10^{-0.1 \alpha D}$ represents the total loss of the transmission line. This then proves that the placement of the amplifier is not a factor in determining the signal power output of a transmission line system so long as internally generated noise is ignored. Indeed, when determining the signal power output of a system for application toward calculation of the

system's SNR, internally generated noise is not used in the analysis. The problem then is to determine the output power of the system noise in order to derive the output SNR.

Assume the noise figure of the amplifier in Figure 3.3 is given as F_a. Using the principles developed in Section G of Chapter 2, the noise figure F of the one amplifier transmission line system is determined to be

$$F = \frac{1}{L_1} + \frac{F_a - 1}{L_1} + \frac{\frac{1}{L_2} - 1}{L_1 A}$$

or

$$F = \frac{F_a L_2 A - L_2 + 1}{L_1 L_2 A} \quad . \qquad \text{(eqn 3.7)}$$

Applying Equation 3.7 to Equation 2.74 it can be determined that the SNR for the one amplifier system is given by

$$\text{SNR} = \frac{P_{in} L_1 L_2 A}{(F_a L_2 A - L_2 + 1 - L_1 L_2 A) k T_0 B} \quad . \qquad \text{(eqn 3.8)}$$

A slightly different approach to the preceding analysis is to determine the total system output noise power by evaluating the noise contribution of each stage of the system. From the discussions on system noise it is apparent that the noise output of the first transmission line segment (P_{n1}) is given by

$$P_{n1} = \left(\frac{1}{L_1} - 1\right) k T_0 B L \quad .$$

This noise is amplified by the amplifier which also generates additional internal noise. Thus the total noise power

at the output of the amplifier (designated P_{n2}) is

$$P_{n2} = (1/L_1 - 1)kT_0BL_1A + (F_a - 1)kT_0BA \, .$$

Similarly this noise power is multiplied by the final stage loss L_2 and added to the thermal noise of the last stage. Therefore the total system noise power is

$$P_{noise} = (\frac{1}{L_1} - 1)kT_0BL_1AL_2 + (F_a - 1)kT_0BAL_2 + (\frac{1}{L_2} - 1)kT_0BL_2$$

which simplifies to

$$P_{noise} = (F_aL_2A - L_1L_2A - L_2 + 1)kT_0B \, .$$

If the input signal power is P_{in} then the output power due to the signal only is $P_{in}L_1L_2A$. Therefore, the system SNR is

$$\text{SNR} = \frac{P_{in}L_1L_2A}{(F_aL_2A - L_1L_2A - L_2 + 1)kT_0B} \, . \qquad \text{(eqn 3.9)}$$

Clearly Equations 3.8 and 3.9 are identical. As a result, the SNR of subsequent systems analysis can be developed by concentrating on the overall system noise figure.

Example 3.2: Assume a transmission line system is to be designed such that only one amplifier is to be inserted in the line. Coaxial cable with an attenuation factor of 0.2dB per kilometer is to be employed to connect a transmitter and receiver which are 100km apart. If the transmitter provides a 1MHz signal at -103dBW, the minimum acceptable SNR is 30dB for signal detection, and the amplifier has 70dB gain and a noise figure of 4dB, the placement of the amplifier can be

determined. Equation 3.9 can be used to quickly solve this problem. First it must be recognized that the product of (L_1) and (L_2) is the total transmission line loss and is 20dB. The "trick" is to isolate L_2 in Equation 3.9 then solve for the lenght of line which yields that particular loss. Letting $L_1 L_2 = L$ or total loss, it can be seen from Equation 3.9 that:

$$F_a L_2 A - L_2 = \frac{P_{in} LA}{(SNR)kT_0 B} + LA - 1$$

so that

$$L_2 = \left[\frac{P_{in} LA}{(SNR)kT_0 B} + LA - 1\right]\left[\frac{1}{F_a A - 1}\right] .$$

Recalling that the variables in Equation 3.9 are ratio values and not dB figures, then substituting the values from the example into this equation yields:

$$L_2 = \left[\frac{(10^{-10.3})(10^{-2})(10^{7})}{(10^{3})(1.379\text{x}10^{-23})(290)(10^{6})} + (10^{5}) - 1\right]\left[\frac{1}{(10^{0.4})(10^{7}) - 1}\right]$$

Or

$$L_2 = 5.38\text{x}10^{-2}$$

Therefore, in dB notation, $L_2 = -12.7$dB. Now since $(L_2)_{dB} = \alpha x$ where x is the length of the transmission line from the amplifier to the receiver and α is -0.2dB/Km, then the lenght of the line is 63.5Km. Therefore, it can be realized

that the amplifier must be inserted no closer than 36.5Km to the transmitter.

For long distance communications via transmission lines, one amplifier inserted into the line will not overcome the losses inherent in the line itself. To solve this problem, multiple amplifiers are inserted so that the typical long distance transmission system resembles Figure 3.4 . In

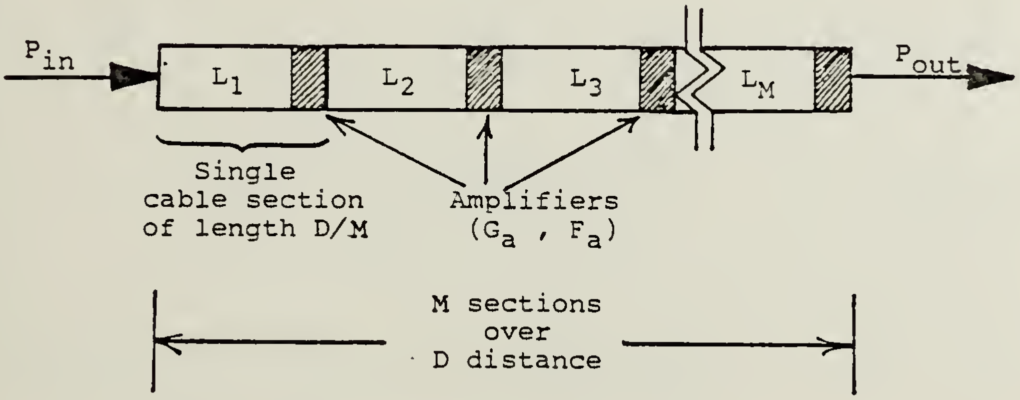

P_in
L_1
L_2
L_3
L_M
P_out
Single cable section of length D/M
Amplifiers (G_a , F_a)
M sections over D distance

Figure 3.4 Cable Model with Multiple Amplifiers.

order to evaluate the overall system output SNR, the noise figure must be calculated. This is most conveniently done by considering the line as a cascade of individual sections, each consisting of an attenuating section of line feeding an amplifier having a specified noise figure.

Consider the cable model of Figure 3.4 containing M identical amplifiers placed equidistant from each other along a line of length D. The cable can be viewed as having M identical sections each of length D/M, and each composed of an attenuating line of loss L and an amplifier of gain A and noise figure F_a. From Equation 2.67 the overall system noise figure is

$$F = \underbrace{\frac{1}{L_1} + \frac{F_a - 1}{L_1}}_{\text{1st stage}} + \underbrace{\frac{\frac{1}{L_1} - 1}{L_1 A} + \frac{F_a - 1}{L_1^2 A}}_{\text{2nd stage}} + \underbrace{\frac{\frac{1}{L_1} - 1}{L_1^2 A^2} + \frac{F_a - 1}{L_1^3 A^2}}_{\text{3rd stage}} + \cdots\cdots + \underbrace{\frac{\frac{1}{L_1} - 1}{L_1^{M-1} A^{M-1}} + \frac{F_a - 1}{L_1^M A^{M-1}}}_{\text{Mth stage}}$$

This can be written as

$$F = \underbrace{\frac{F_a}{L_1}}_{1} + \underbrace{\frac{\frac{1}{L_1} - 1}{L_1 A} + \frac{\frac{F_a}{L_1} - \frac{1}{L_1}}{L_1 A}}_{2} + \underbrace{\frac{\frac{1}{L_1} - 1}{L_1^2 A^2} + \frac{\frac{F_a}{L_1} - \frac{1}{L_1}}{L_1^2 A^2}}_{3} + \cdots\cdots + \underbrace{\frac{\frac{1}{L_1} - 1}{L_1^{M-1} A^{M-1}} + \frac{\frac{F_a}{L_1} - \frac{1}{L_1}}{L_1^{M-1} A^{M-1}}}_{M}$$

or

$$F = \underset{1}{\frac{F_a}{L_1}} + \underset{2}{\frac{\frac{F_a}{L_1} - 1}{L_1 A}} + \underset{3}{\frac{\frac{F_a}{L_1} - 1}{L_1^2 A^2}} + \qquad + \underset{M}{\frac{\frac{F_a}{L_1} - 1}{L_1^{M-1} A^{M-1}}}$$

so that

$$F = \frac{F_a}{L_1} + \sum_{k=2}^{M} \frac{(F_a/L_1) - 1}{(L_1 A)^{k-1}} \; . \qquad \text{(eqn 3.10)}$$

But F_a/L_1 can be viewed as $[(F_a/L_1) - 1] + 1$. Then Equation 3.10 can be written as follows:

$$F = 1 + \sum_{k=1}^{M} \frac{(F_a/L_1) - 1}{(L_1 A)^{k-1}}$$

or

$$F = 1 + \left(\frac{F_a}{L_1} - 1\right) \sum_{k=1}^{M} \left(\frac{1}{L_1 A}\right)^{k-1} . \qquad \text{(eqn 3.11)}$$

Since

$$\sum_{k=1}^{M} \left(\frac{1}{L_1 A}\right)^{k-1} = \sum_{k=0}^{M-1} \left(\frac{1}{L_1 A}\right)^{k}$$

from the well-known closed form sum for a geometric series given on page 87, Equation 3.11 can be expressed in closed

form so that the overall system noise figure for a typical long distance transmission line system becomes

$$F = \begin{cases} 1 + \left(\frac{F_a}{L_1}\right) - 1 \; \dfrac{1^{-\left(\frac{1}{L_1 A}\right)}}{1 - \frac{1}{L_1 A}} & \text{for } L_1 A \neq 1 \quad \text{(eqn 3.12)} \\ \\ 1 = \left(\frac{F_a}{L_1} - 1\right) M & \text{for } L_1 A = 1 \,. \quad \text{(eqn 3.13)} \end{cases}$$

If the gain of the amplifier used in each subsection of the transmission line exactly compensates for the line loss of the subsection then $L_1 \; A = 1$ and Equation 3.13 must be used to determine system noise figure. Otherwise Equation 3.12 must be used. Then, depending on the line loss and amplifier gain relationship, either Equation 3.12 or 3.13 is used together with the result of Equation 2.74 in order to determine the total system's output SNR.

Example 3.3: A transmission line with an attenuation index of 0.5 dB/Km is used to provide a communications link for a transmitter and receiver 100Km apart. Identical amplifiers whose individual noise figure is 3dB, are inserted in equidistant form along the line such that each amplifier gain equals the line attenuation over each segment of the line. Assume a 100KHz information signal is placed on the line at an input power level of 1microwatt and assume that the minimum acceptable SNR at the receiver is 74dB. The maximum allowable system noise figure, minimum number of amplifiers, and amplifier spacing can be determined from Equations 2.74 and 3.13 as follows:

$$F = \frac{P_{in}}{(SNR)kT_0B} + 1$$

$$F = \frac{10^{-6}}{(10^{7.4})(1.379x10^{-23})(290)(10^{5})} + 1$$

$$F = 100$$

so that the system noise figure is 20dB. Then from Equation 3.13

$$M = [100 - 1]/[(F_a/L_1) - 1],$$

But from Equation 3.3

$$L_1 = 10^{-0.1\alpha\frac{D}{M}}$$

so that M must be determined by trial and error. The following table was developed to determine the minimum number of amplifiers used to achieve a noise figure of 100. Table III clearly indicates that the minimum number of amplifiers which yields a system noise figure at or below the 20dB allowable limit is 5.

TABLE III

M Determination via Trial and Error

M	L	Noise Figure
2	0.003	31.0 dB
3	0.02	24.0 dB
4	0.05	21.0 dB
5	0.10	19.8 dB
6	0.15	18.8 dB
8	0.24	17.8 dB

Transmission lines can be analyzed with the aid of circuit theory - that is, in terms of voltages, currents, and impedances. Due to the distributed nature of impedances in a transmission line, a signal voltage applied at the input end of a transmission line at a certain instant of time (for example, by closing a switch) will not appear instantaneously at the output end of the line. It will travel along the line at some finite velocity which is that of the electromagnetic field associated with the voltage and current of the line. Because time is required for a signal waveform to travel the length of a transmission line, the output voltage will not be in phase with the input voltage, and the output current will not be in phase with the input current. Thus, signals experience a time delay and phase shift as they propagate along the length of the line. Furthermore, as indicated earlier in this section, the magnitudes of the voltage and current at the output terminals of a line will not be the same as the magnitudes of the voltage and current at the input terminals due to the line's attenuation.

The magnitudes of the attenuation and phase shift that occur are determined by the propagation constant of the transmission line. The propagation constant, represented by γ, is a complex number with the real part being the attenuation constant (denoted ψ) and the imaginary part being the phase constant (denoted β).

Earlier portions of this chapter were concerned with a generalized approach to system analysis of transmission lines in which the gross behavior of the line was characterized by the loss parameter α. Consequently, the attenuation constant of a line was treated as a given parameter since it can be obtained from manufacturing specifications. Furthermore, for the generalized analysis approach to system performance, the details of line attenuation and phase/time

delay are inconsequential. The remaining portions of this chapter will concentrate on developing an overall understanding of line attenuation and phase/time delay effects.

B. LOW FREQUENCY MODEL

While the general study of communication system transmission lines is primarily concerned with high frequency signals, it is beneficial to first develop an understanding of transmission line characteristics that are applicable at lower frequencies and then apply that knowledge to a model that is applicable at higher frequencies. It has been shown [Ref. 29] that a transmission line can be analyzed in terms of AC circuit theory by obtaining an equivalent circuit of the line. This is possible because a transmission line is a distributed parameter device and can therefore be described as a cascade of incremental networks of lumped resistive, capacitive, and inductive elements. Consider a transmission line composed of two long parallel wires suspended in air such that a small subsection of length x can be examined [Figure 3.5]. One restriction placed on the equivalent circuit is that the length x of each subsection must be smaller than the wavelength of the applied signal frequency. Then each subsection can be considered a lumped circuit and the various elements within the subsection defined.

The equivalent circuit of a transmission line is composed of a series resistance (R), a series inductance (L), a shunt conductance (G), and a shunt capacitance (C). It should be noted that the shunt conductance is the leakage conductance (inverse of resistance) of the dielectric material (including air) placed between the conducting wires and not directly related to the resistance R of the conducting wire. Line parameters are normally given as per-unit-length values, that is, R in ohms per unit length,

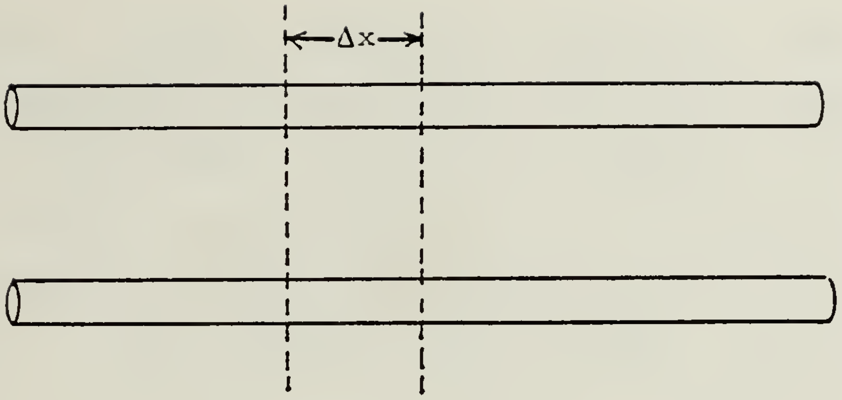


Figure 3.5 Parallel Wire Transmission Line.

L in henries per unit length, C in farads per unit length, and G in mhos per unit length. To derive the value of the equivalent circuit components, it is necessary to multiply these line parameters by Δx, the length of the subsection. The equivalent circuit can therefore be modeled as Figure 3.6 .

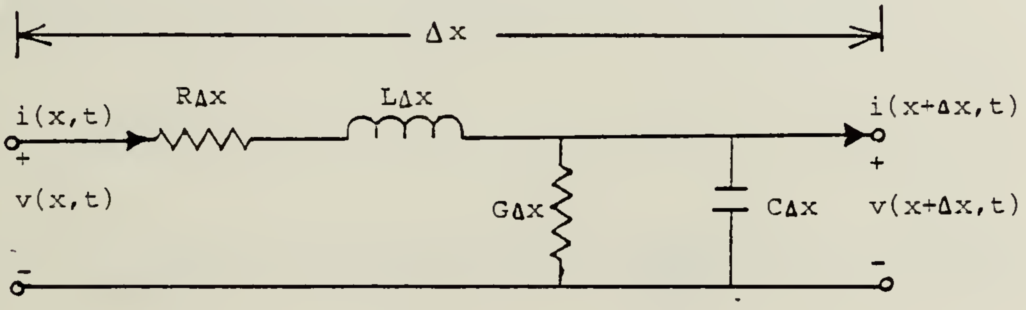


Figure 3.6 Equivalent Circuit.

While all transmission line theory could be treated in terms of ac circuit analysis, the analyses would be extremely involved for all but the simple cases [Ref. 30]. It is more convenient to treat transmission lines in terms of differential equations (see Appendix A for a general description of differential equations). The differential equations are obtained from a simple ac circuit analysis of

the equivalent circuit of line subsection and then by letting the incremental section of line (Δx) approach zero. Consider the equivalent circuit of Figure 3.6 where the current and voltage are functions of both position and time. It is evident that voltage and current are in general functions of time. Line current and voltage are functions of position due to the finite velocity of propagation of the signal waveform. For a line segment of length Δx which begins at the arbitrary point x, input and output voltages are denoted $v(x,t)$ and $v(x+\Delta x,t)$ respectively. Similarly, input current is $i(x,t)$ and output current is $i(x+\Delta x,t)$.

Since the voltage drop across the resistor is $R\Delta x i(x,t)$ and across the inductor it is $L\Delta x\frac{\partial}{\partial t}i(x,t)$, then Kirchhoff's voltage law can be applied to the equivalent circuit to yield

$$v(x,t) - R\ xi(x,t) - L\ x\frac{\partial}{\partial t}i(x,t) - v(x+\Delta x,t) = 0$$

or

$$v(x+\Delta x,t) - v(x,t) = -R\ xi(x,t) - L\ x\frac{\partial}{\partial t}i(x,t)\ . \quad \text{(eqn 3.14)}$$

Likewise, Kirchhoff's current law can be applied to the circuit. Noting that current flow into the equivalent circuit capacitor is $C\Delta x\frac{\partial}{\partial t}v(x+\Delta x,t)$ and that the current flow through the shunt resistor is $G\Delta x v(x+\Delta x,t)$, then

$$i(x,t) - G\ xv(x+\Delta x,t) - C\ x\frac{\partial}{\partial t}v(x+\Delta x,t) - i(x+\Delta x,t) = 0$$

or

$$i(x+\Delta x,t) - i(x,t) = -G\,x\,v(x+\Delta x,t) - C\,x\frac{\partial}{\partial t}v(x+\Delta x,t) \quad . \text{(eqn 3.15)}$$

Dividing Equations 3.14 and 3.15 by Δx results in

$$\frac{v(x+\Delta x,t) - v(x,t)}{\Delta x} = -Ri(x,t) - L\frac{\partial}{\partial t}i(x,t)$$

and

$$\frac{i(x+\Delta x,t) - i(x,t)}{\Delta x} = -Gv(x+\Delta x,t) - C\frac{\partial}{\partial t}v(x+\Delta x,t)$$

respectively. Then in the limit as Δx approaches zero, these equations become:

$$\frac{\partial v(x,t)}{\partial x} = -Ri(x,t) - L\frac{\partial i(x,t)}{\partial t} \quad \text{(eqn 3.16)}$$

and

$$\frac{\partial i(x,t)}{\partial x} = -Gv(x,t) - C\frac{\partial v(x,t)}{\partial t} \quad . \quad \text{(eqn 3.17)}$$

It can be seen that Equations 3.16 and 3.17 are of analogous form. As pointed out in [**Ref. 31**], these equations are the dual of one another with the following analogous quantities:

$v \longleftrightarrow i$ $\qquad$ $R \longleftrightarrow G$ $\qquad$ $L \longleftrightarrow C$

as also evidenced from equations 3.14 and 3.15 . Equations 3.16 and 3.17 have become known as telegraphist's equations and are the basis for all parametric analysis of transmission lines [Ref. 32]. The task then is to solve these two equations.

For initial analysis consider a lossless line such that $R = G = 0$. Then

$$\frac{\partial}{\partial x} v(x,t) = -L \frac{\partial}{\partial t} i(x,t)$$

and

$$\frac{\partial}{\partial x} i(x,t) = -C \frac{\partial}{\partial t} v(x,t)$$

which, as will be demonstrated, are solved jointly. However lines are not lossless; in fact, since lines are seldom coiled the associated line inductance (L) and conductance (G) are typically small. It is therefore reasonable to approximate L and G by zero [Ref. 33] so that the telegraphist's equations become

$$\frac{\partial}{\partial x} v(x,t) = -R\, i(x,t) \qquad \text{(eqn 3.18)}$$

and

$$\frac{\partial}{\partial x} i(x,t) = -c \frac{\partial}{\partial t} v(x,t) \quad . \qquad \text{(eqn 3.19)}$$

These equations can be solved assuming ac steady state conditions on the line. This is obtained by assuming that v(x,t) consists of a sinusoidal time dependent form given by

$v(x,t) = V(x)e^{j\omega t}$. Because of the duality of Equations 3.18 and 3.19, it can be guaranteed that $i(x,t)$ will be of the form $I(x)e^{j\omega t}$ where $V(x)$ and $I(x)$ are in general complex functions of x and referred to as phasors in ac circuit theory. By differentiating $v(x,t)$ and $i(x,t)$ with respect to x yields:

$$\frac{\partial}{\partial x} v(x,t) = \frac{d\,V(x)}{dx}\, e^{j\omega t}$$

and

$$\frac{\partial}{\partial x} i(x,t) = \frac{d\,I(x)}{dx}\, e^{j\omega t}$$

respectively. Using these equations for a steady state analysis of Equations 3.16 and 3.17 gives:

$$\frac{d\,V(x)}{dx}\, e^{j\omega t} = -RI(x)e^{j\omega t} - LI(x)j\, e^{j\omega t}$$

and

$$\frac{d\,I(x)}{dx}\, e^{j\omega t} = -GV(x)e^{j\omega t} - CV(x)j\, e^{j\omega t}\ .$$

It can be easily seen that $e^{j\omega t}$ factors out of both equations so that:

$$\frac{d\,V(x)}{dx} = -RI(x) \quad -LI(x)j\omega \qquad \text{(eqn 3.20)}$$

and

$$\frac{d\,I(x)}{dx} = -GV(x) \;\; -CV(x)j\omega \qquad \text{(eqn 3.21)}$$

which can be solved simultaneously. Equation 3.21 can be written as

$$\frac{d\,I(x)}{dx} = [-G \;\; -Cj\omega]V(x)$$

so that the second derivative of I(x) is

$$\frac{d^2V(x)}{dx^2} = [-G \;\; -Cj\omega]\,\frac{d\,V(X)}{dx}$$

which can equivalently be put in the form:

$$\frac{d\,V(x)}{dx} = \frac{1}{-G \;\; -Cj\omega}\,\frac{d^2I(x)}{dx^2} \quad .$$

Substituting this into Equation 3.20 yields:

$$\frac{1}{-G \;\; -Cj\omega}\;\frac{d^2I(x)}{dx^2} = -RI(x) \;\; -LI(x)j\omega$$

or

$$\frac{d^2I(x)}{dx^2} = (G + Cj\omega)(R + Lj\omega)\;I(x). \qquad \text{(eqn 3.22)}$$

Letting

$$\gamma^2 = (G + Cj)(R + Lj) \qquad \text{(eqn 3.23)}$$

Equation 3.22 becomes:

$$\frac{d^2 I(x)}{dx^2} = \gamma^2 I(x) \; . \qquad \text{(eqn 3.24)}$$

This is a simple linear differential equation of the second order for which a solution is well known [Ref. 34].

It can be shown that

$$I(x) = I_A e^{-\gamma x} + I_B e^{\gamma x} \qquad \text{(eqn 3.25)}$$

where I_A and I_B are constants of integration determined by the boundary conditions at the input and output ends of the transmission line. Again, because of the duality of the telegraphist's equations it can be demonstrated that

$$\frac{d^2 V(x)}{dx} = \gamma^2 V(x)$$

so that

$$V(x) = V_A e^{-\gamma x} + V_B e^{\gamma x} \qquad \text{(eqn 3.26)}$$

and the constants of integration V_A and V_B are again related to line boundary conditions [Ref. 35]. From Equations 3.20 and 3.26 it can be shown that the current at any point x along the line in terms of the constants V_A and V_B is:

$$I(x) = \sqrt{\frac{G + j L}{R + j C}} \left(V_A e^{-\gamma x} - V_B e^{\gamma x} \right).$$

Letting

$$Z = R + j\omega L \quad \text{and} \quad Y = G + j\omega C$$

this equation becomes

$$I(x) = \frac{1}{\sqrt{Z/Y}} (V_A e^{-\gamma x} - V_B e^{\gamma x}) \; . \qquad \text{(eqn 3.27)}$$

Substituting for I(x) from Equation 3.25 yields

$$I_A e^{-\gamma x} + I_B e^{\gamma x} = \frac{1}{\sqrt{Z/Y}} (V_A e^{-\gamma x} - V_B e^{\gamma x})$$

so that

$$I_A = - \frac{V_A}{\sqrt{Z/Y}}$$

and

$$I_B = - \frac{V_B}{\sqrt{Z/Y}} \; .$$

Therefore, the voltage and current at any point along the line can be written in terms of the constants V_A and V_B as:

$$v(x,t) = (V_A e^{-\gamma x} + V_B e^{\gamma x}) e^{j\omega t} \qquad \text{(eqn 3.28)}$$

and

$$i(x,t) = \left[\frac{V_A}{\sqrt{Z/Y}} e^{-\gamma x} - \frac{V_B}{\sqrt{Z/Y}} e^{\gamma x} \right] e^{j\gamma t} . \qquad \text{(eqn 3.29)}$$

V_A and V_B are determined by the boundary conditions of the input and output ends of the transmission line and therefore require further investigation.

1. Characteristic Impedance

Assume a transmission line of length ℓ that has as its input a voltage source V_s with internal impedance Z_s . Also assume the line has a load impedance Z_L connected to its output terminal (see Figure 3.7). The voltage and current phasors at the input end of the line (x=0) are designated V_0 and I_0 while the line output voltage and current phasor are V_ℓ and I_ℓ respectively. Using Kirchhoff's Voltage law it can be shown that at x=0

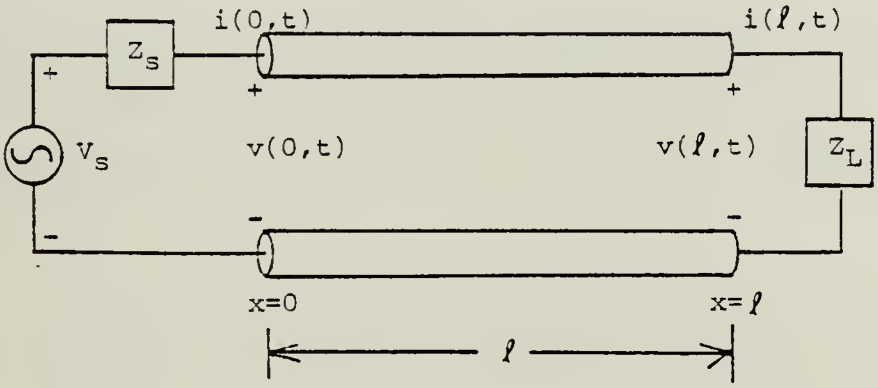


Figure 3.7 Electrical Elements of the Transmission Line.

$$V_s - Z_s I_0 - V_0 = 0 \qquad \text{(eqn 3.30)}$$

and at $x = \ell$

$$V_\ell - Z_L I_\ell = 0 \, .$$

This equation indicates that

$$Z_L = \frac{V_\ell}{I_\ell}$$

which can be written as

$$Z_L = \frac{V_A \, e^{-\gamma\ell} + V_B \, e^{\gamma\ell}}{\frac{V_A}{\sqrt{Z/Y}} \, e^{-\gamma\ell} - \frac{V_B}{\sqrt{Z/Y}} \, e^{\gamma\ell}} \, . \qquad \text{(eqn 3.31)}$$

For simplicity let $Z_0 = \sqrt{Z/Y}$. Furthermore, for maximum power transfer from the source to the line (i.e. impedance matching conditions) let $Z_s = Z_0$. Then from Equations 3.26, 3.27, and 3.30

$$V_s - Z_s \left[\frac{V_A}{Z_0} e^{-0} - \frac{V_B}{Z_0} e^{0} \right] - \left[V_A \, e^{-0} + V_B \, e^{0} \right] = 0$$

which yields

$$V_A = \frac{V_s}{2} \, . \qquad \text{(eqn 3.32)}$$

Substitution this value for V_A into Equation 3.31, it can be seen that:

$$Z_L = Z_0 \left[\frac{\frac{1}{2} V_s \, e^{-\gamma\ell} + V_B \, e^{\gamma\ell}}{\frac{1}{2} V_s \, e^{-\gamma\ell} - V_B \, e^{\gamma\ell}} \right]$$

so that

$$V_B = \frac{V_s}{2} e^{-2\gamma \ell} \left[\frac{Z_L - Z_0}{Z_L + Z_0} \right] . \qquad \text{(eqn 3.33)}$$

Now from Figure 3.7 it can be said that the input impedance (Z_{in}) at x=0 is given by

$$Z_{in} = \frac{V_0}{I_0}$$

and substituting Equations 3.26 and 3.27 with x set to zero yields

$$Z_{in} = \frac{V_A + V_B}{\frac{V_A}{Z_0} - \frac{V_B}{Z_0}}$$

where Z_0 is $\sqrt{Z/Y}$. Further, substituting Equations 3.32 and 3.33 for V_A and V_B gives

$$Z_{in} = Z_0 \left[\frac{\frac{V_s}{2} + \frac{V_s}{2} e^{-2\gamma \ell} \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right)}{\frac{V_s}{2} - \frac{V_s}{2} e^{-2\gamma \ell} \left(\frac{Z_L - Z_0}{Z_L + Z_0} \right)} \right]$$

which can be simplified to

$$Z_{in} = Z_0 \left[\frac{1 + e^{-2\gamma \ell} \, [(Z_L - Z_0)/(Z_L + Z_0)]}{1 - e^{-2\gamma \ell} \, [(Z_L - Z_0)/(Z_L + Z_0)]} \right] . \qquad \text{(eqn 3.34)}$$

For a very long line (i.e. $\ell \longrightarrow \infty$) $e^{-2\gamma\ell} \longrightarrow 0$ so that

$$Z_{in} = Z_0 \, \frac{1 + 0}{1 - 0} = Z_0 \quad . \qquad \text{(eqn 3.35)}$$

Therefore the input impedance of the line is Z_0 which from a previous definition, can be seen to be given by

$$Z_0 = \frac{R + j\,L}{G + j\,C} \qquad \text{(eqn 3.36)}$$

This is the so-called characteristic impedance of the line, since it depends on the characteristic parameters R, L, G, and C. It can be seen that for the general case the impedance is complex and is a function of the applied frequency ω. There are two cases, however, when the characteristic impedance is not a function of frequency.

First consider a lossless line where R and G equal zero. Then $Z_0 = \sqrt{L/C}$ which is a pure resistance, independent of frequency (so long as L and C are themselves frequency independent). A second case occurs when L/R = C/G, that is when the time constants of the equivalent circuit are equal. So that

$$Z_0 = \sqrt{\frac{1 + j\omega\frac{L}{R}}{1 + j\omega\frac{C}{G}}} = 1$$

when L/R = C/G [Ref. 36].

With the knowledge that the impedance of a line is determined by its characteristic parameters Z_0, in accordance with Equation 3.36, the analyst can now focus on the question, "If the impedance of the line is determined by its physical characteristics, then how does the characteristic impedance affect the signal as it propagates down the line?"

2. Propagation Constant

The comparison of a system's output signal to its input signal is a measure of that system's effect on the signal and, as discussed in Chapter 1, is generally described by the system's transfer function [H(ω)]. This principle can be used to derive the relationship between the characteristics of the line and its effect on the propagating signal.

Consider the line in Figure 3.7 whose characteristic impedance Z_0 and load impedance Z_L are matched. Then, by Equation 3.33 for $Z_0 = Z_L$, $V_B = 0$ so that from Equation 3.28 $v(x,t) = \frac{1}{2} V_S\, e^{-\gamma x} e^{j\omega t}$. Therefore, the ratio of the signal voltage at the output side of the line to the signal voltage at the input side of the line is given by:

$$\frac{v(\ell,t)}{v(0,t)} = \frac{\frac{V_s}{2} e^{-\gamma} \quad e^{j\omega t}}{\frac{V_s}{2} e^{-0} \quad e^{j\omega t}} = e^{-\gamma\ell}$$

which can be written as:

$$V(x=\ell) = V(x=0)e^{-\gamma\ell} . \qquad \text{(eqn 3.37)}$$

In other words, the signal voltage at the output end of the line is equal to the signal voltage at the input side of the line times $e^{-\gamma\ell}$ which, by definition [page 17], is the transfer function of the line and designated $H(\omega)$. Earlier in the analysis the definition $\gamma^2 = (R+j\omega L)(G+j\omega C)$ was made so that:

$$\gamma = \sqrt{(R + j\ L)(G + j\ C)} \qquad \text{(eqn 3.38)}$$

which is a complex quantity and can be written in the form

$$\gamma = \psi + \beta j$$

where ψ is the real part of γ and β is the imaginary part of γ. Combining these definitions, it can be seen that

$$e^{-\gamma \ell} = H(\ ) = e^{-(\psi + \beta j)\ell}$$

so that

$$H(\omega) = e^{-\psi \ell} e^{-j\beta \ell} . \qquad \text{(eqn 3.39)}$$

As a result, it is easily seen that by its transfer function, the transmission line introduces both signal attenuation ($e^{-\psi\ell}$) and delay (which is related to the term $e^{-j\beta\ell}$). Furthermore, as the length of the line increases so do the attenuatuion and delay. The term γ is called the line's propagation constant and is a per unit length value determined from the characteristic parameters of the line.

Initial sections of this chapter introduced the transmission line attenuation factor (α) as a gross quantity per unit length of line so that the perspective of the general transmission line analysis could be focused on the system and not on component level details. From the simplification of Equation 3.23 , the attenuation factor of a transmission line can be deduced. If $\gamma^2 = (R + j\omega L)(G + j\omega L)$ then γ^2 is a complex quantity equal to

$$(RG - \omega^2 LC) + j\omega(LG + RC)$$

which is of the form a + jb. As demonstrated in the foregoing section $\gamma = \psi + j\beta$, then:

$$\gamma^2 = \psi^2 - \beta^2 + j(2\psi\beta)$$

where $\psi^2 - \beta^2$ is the real part of γ^2 and $2\psi\beta$ is the imaginary part of γ^2. Therefore letting

$$a = \psi^2 - \beta^2$$

and

$$b = 2\psi\beta$$

The second equation can be written as $\beta = b/(2\psi)$ so that β can can be substituted into the first equation to yield:

$$\psi^4 - a\psi^2 - \frac{1}{4}b^2 = 0 \ .$$

Using the general solution for a quadratic equation gives:

$$\psi^2 = \frac{a \pm \sqrt{a^2 + b^2}}{2}$$

so that

$$\psi = \left[\frac{a \pm \sqrt{a^2 + b^2}}{2}\right]^{1/2}$$

Now substituting (RG $-\omega^2$LC) for a and (LG + RC) for b it can be seen that :

$$\psi = \left[\frac{RG - \omega^2 LC + \sqrt{(RG - {}^2 LG)^2 + \omega^2 (LG + RG)^2}}{2} \right]^{1/2} \qquad \text{(eqn 3.40)}$$

which determines the attenuation constant of a unit length line. It should be noted that while the solution for a quadratic equation include both plus and minus square root values, the positive value only is used to determine ψ. Clearly, the absolute value of the square root term will always exceed $|RG - \omega^2 LC|$ so that if the negative square root were considered ψ would be purely imaginary. Since attenuation is purely real, the negative square root term is meaningless. Then, substituting the value of ψ from Equation 3.40 into the equation equation $\beta = b/(2\psi)$ and replacing b with LG + RC it can be seen that

$$\beta = \frac{\omega(LG + RC)}{\left[2RG - 2\omega^2 LC + 2\sqrt{(RG - \omega^2 LC)^2 + \omega^2 (LG + RC)^2} \right]^{1/2}} \cdot \qquad \text{(eqn 3.41)}$$

This term β then describes the delay constant of the unit length line. It should be noted that the attenuation constant and the delay constant are dependent on both the line operating frequency and the line characteristic parameters.

Earlier in this chapter a transmission line was grossly characterized as "lossy" where its total line attenuation was found by multiplying the length of the line by the line's attenuation factor α which could either be measured or stated in the manufacturer's specifications. The question then is, "What is the relationship between the line's propagation constant and the transmission power loss?"

3. Power Loss

Referring back to Figure 3.7 where input and output impedances are matched, the voltage and current due to a sinusoidal input at any point x along the line are (with time dependency suppressed) given by:

$$V(x) = \frac{V_s}{2} e^{-\gamma x}$$

and

$$I(x) = \frac{V_s}{2Z_0} e^{-\gamma x}$$

respectively. It can be demonstrated that power at any point x along the line is given by the voltage at x times the complex conjugate of the current at x [Ref. 37]. This is then written as

$$P(x) = V(x) I^*(x)$$

where * indicates complex conjugation [Ref. 38]. It can now be shown that power at the input end of the (x = 0) is:

$$P_{in} = P_0 = V_0 I_0^* = \left(\frac{V_s}{2}\right)^2 \left(\frac{1}{Z_0}\right)^*$$

and that power at the output end of the line (x = ℓ) is:

$$P_{out} = P_\ell = V_\ell I_\ell = \left(\frac{V_s}{2}\right)^2 \left(\frac{1}{Z_0}\right)^* \left(e^{-\gamma \ell}\right)\left(e^{-\gamma \ell}\right)^* .$$

As previously discussed in this thesis, power loss is simply a comparison or ratio of output power to input power; therefore a transmission line's power loss can be written as:

$$\frac{P_{out}}{P_{in}} = \frac{P_{\ell}}{P_o} = (e^{-\gamma\ell})(e^{-\gamma\ell})^*$$

Recalling that $\gamma = \psi + j\beta$, then

$$\frac{P_{out}}{P_{in}} = e^{-(\psi+j\beta)\ell}\, e^{-(\psi-j\beta)\ell}$$

so that

$$\frac{P_{out}}{P_{in}} = e^{-2\psi\ell}$$

is the power loss over the length ℓ of a line. But, power loss (or attenuation) is normally given in dB's, therefore

$$\left(\frac{P_{out}}{P_{in}}\right)_{dB} = 10 \log e^{-2\psi\ell}$$
$$= 20\psi\ell \log e$$

where e is the base of the natural logarithm and has the value 2.71828.... Therefore, the dB attenuation or loss (L) of a transmission line of length ℓ is given by

$$(L)_{dB} = 8.6858896\,\psi\ell\,.$$

It can now be seen that the unit length ($\ell = 1$) transmission line attenuation factor α, earlier presented as a gross quantity, is determined by:

$$\alpha = 8.69\psi \qquad \text{(eqn 3.42)}$$

where ψ is determined from Equation 3.40. More appropriately termed the line's attenuation constant, α is a function of the line's characteristic parameters and its operating frequency. It should be noted that the delay constant has no effect on power as the signal propagates along the line.

4. Reflection Coefficients

The previous sections of this chapter were developed on the basis of the load impedance and the line characteristic impedance being of the same value. Under these conditions the entire voltage and current waveforms propagate down the transmission line with no reflection from the load when $\ell \longrightarrow \infty$. It was also shown that with $Z_0 = Z_L$ the V_B and I_B terms in the respective line voltage and current equations were zero. However, when a mismatch between the load and line impedances exists, the V_B and I_B terms do not equate to zero and therefore detract by some factor from the waveforms transmitted to the load. These detracting factors are known as reflection coefficients and represent an apparent waveform traveling from the output end of the transmission line to its input end.

The voltage reflection coefficient is denoted ρ_V and is defined as:

$$\rho_V \triangleq \frac{V_B \, e^{\gamma\ell}}{V_A \, e^{-\gamma\ell}} . \qquad \text{(eqn 3.43)}$$

Substituting the value for V_A and V_B as previously derived [Equation 3.32 and 3.33] the voltage reflection coefficient becomes

$$\rho_v = \frac{Z_L - Z_0}{Z_L + Z_0} \quad . \qquad \text{(eqn 3.44)}$$

From Equation 3.43 it can be seen that V_B will be related to V_A in the following manner:

$$V_B = \rho_v V_A e^{-2\gamma\ell} \; .$$

The generalized equation for voltage at any point x along a transmission line of finite length ℓ is:

$$v(x,t) = \frac{V_s}{2}\left[e^{-\gamma x} + {}_v e^{\gamma(x-2\ell)}\right] e^{j\omega t} \; .$$

The current reflection coefficient, denoted ρ_i, is similary defined as:

$$\rho_v \triangleq \frac{I_B e^{\gamma\ell}}{I_A e^{-\gamma\ell}}$$

and from the current-voltage relationship previously developed it can be shown that $\rho_i = -\rho_v$. Therefore, given the voltage reflection coefficient ρ_v, the generalized current equation at any point x along a line of length can be stated by the equation:

$$i(x,t) = \frac{V_s}{2Z_0}\left[e^{-\gamma x} - \rho_v e^{\gamma(x-2\ell)}\right] e^{j\omega t}.$$

5. Transmission Line Characteristic Parameters

The beginning of this section on the low frequency model briefly considered the parameters required to construct the equivalent circuit of a transmission line. These equivalent circuit component values are a function of material composition and line cross-sectional geometry. Rather than developing an analysis of the characteristic parameter of a line, this section simply presents a summary of the standard coaxial cable and open, two-wire models derived in [Ref. 39]. For both models, resistance and conductance are related to the material used for the conducting wire and the insulation respectively. Referring to Figure 3.8 , the resistance (R) of a line is equal to the resistivity ($1/\sigma$) of the conducting material divided by its cross-sectional area. Thus

$$R = \frac{1/\sigma}{r^2}$$

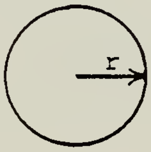


Figure 3.8 Circular Wire.

Referring to Figure 3.9 or 3.10 conductance (G) is the inverse of the resistivity ($1/\sigma$) of the material separating the two conducting wires. Therefore:

$$G = \sigma \ .$$

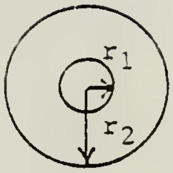


Figure 3.9 Coaxial Cable Model.

a. Coaxial Cable

Figure 3.9 depicts the cross-sectional geometry of a coaxial cable. Capacitance (C) can be calculated using the equation:

$$C = \frac{2\pi\epsilon}{\ln(r_2/r_1)}$$

where r_1 and r_2 represent the inner radii of the two conductors, and ϵ is the permittivity of the insulating material. As an aside, the permittivity of a vacuum (denoted ϵ_0) is 8.85×10^{-12} farads/meter. The inductance (L) of a coaxial cable can be calculated from the equation:

$$L = \frac{\mu}{4\pi}\left[\frac{1}{2} + 2\ln(r_2/r_1)\right]$$

where μ is the permeability of the insulating material. If the two conductors are separated by air the permeability of a vacuum ($\mu_0 = 4\pi\times10^{-7}$ henries/meter) can be used as a close approximation [Ref. 40].

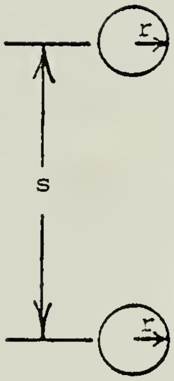


Figure 3.10 Open, Two-Wire Model.

b. Open, Two-Wire Line

The cross-sectional geometry of the open, two-wire model assumes that the conducting wires are of equal size as illustrated in Figure 3.10. The radius of each conducting wire is r and s is the distance separating the centers of the wires. Then the capacitance of the line is given by:

$$C = \frac{\pi \epsilon}{\ln\left(\frac{s + \sqrt{s^2 - 4r^2}}{2r}\right)}$$

where ϵ is the permittivity of the material between the two wires. Inductance is given by

$$L = \frac{\mu}{\pi} \ln\left(\frac{s + \sqrt{s^2 - 4r^2}}{2r}\right)$$

where μ is the permeability of the insulating material between the conductors.

C. SKIN EFFECT MODEL

The low frequency model for transmission lines assumes that a line's characteristic parameters are uniformly distributed throughout the line. It has been demonstrated that γ is a function of the line's parameters and operating frequency but that the effects of frequency are negligible when compared with the line resistance and dielectric conductance at the lower frequency spectrum. Therefore, current was assumed to be uniformly distributed through the cross-sectional and longitudinal areas of a line. It is known that when an alternating current flows in a conductor, the associated magnetic flux within the conductor induces an electromotive force (EMF). This EMF causes the current density to decrease at the center of a wire and increase at the outer surface. This migration of current toward the surface of a conductor is known a the skin effect and increases in prominence as the frequency of the input signal increases. For transmission lines operating at low frequencies, skin effect is negligible and, for all practical purposes, can be ignored; however, for lines operated at higher frequencies, skin effect becomes a significant factor [Ref. 41].

The skin effect model, which was developed for transmission lines operated at high frequencies, accounts for the skin effect phenomenon. As current migrates toward the surface of a conductor, the effective resistance of the conductor increases. For high frequency transmission lines, the line series resistance (R) is frequency dependent. The telegraphist's equations are the basis for all transmission line analysis regardless of operating frequencies. Since the general concepts and equations for transmission lines have been already presented, the skin effect model can be easily analyzed by the use of the already developed model and equations.

The objective of a transmission line analysis is to determine the line's effect on an input signal. As demonstrated in the low frequency model, a line's transfer function is dependent only on its propation constant (γ) and the length (ℓ) of the line. From the analysis of the low frequency model, the propagation constant of a line operating at high frequencies is [from Equation 3.38]:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

where R now is the skin effect impedance Z_{SK}. The skin effect impedance has been found to be (approximately)

$$Z_{SK} = K\sqrt{j\omega}$$

where K is the skin effect resistance and ω is the line operating frequency. At high frequencies, a circular conducting wire has a skin effect resistance given by:

$$K = \frac{1}{2\pi r}\sqrt{\mu/\sigma}$$

where r is the radius of the wire, μ is the permeability of the wire, and $1/\sigma$ is the resistivity of the wire [Ref. 42]. Letting $p = \sqrt{j\omega}$ and substituting for R, the propagation constant for a high frequency line is:

$$\gamma = \sqrt{(K\sqrt{p} + pL)(G + pC)} \quad .$$

In a similar manner the characteristic impedance of a line operating at high frequencies can be determined using the Z_{SK} value in place of the R term in Equation 3.36 Therefore, for the skin effect model, a transmission line's characteristic impedance is given by:

$$Z_0 = \sqrt{\frac{K\sqrt{p} + pL}{G + pC}} .$$

For all practical purposes, the conductance of line insulation between the two conductors is negligible [Ref. 43] so that the equation for the propagation constant of a high frequency line is

$$\gamma = \sqrt{(K\sqrt{p} + pL)(pC)} \qquad \text{(eqn 3.45)}$$

and the characteristic impedance equation is

$$Z_0 = \sqrt{\frac{K\sqrt{p} + pL}{pC}} . \qquad \text{(eqn 3.46)}$$

Given an operating frequency and the line parameters, Equations 3.45 and 3.46 can be solved and applied to all other equations presented in the low frequency model section. The remaining portions of this chapter will concentrate on the high frequency line's transfer function, its effect on applied digital signals, and the expected output signal-to-noise ratio under certain specified conditions.

1. Transfer Function

In the low frequency model it was demonstrated that the transfer function $H(\omega)$ of a transmission line of length ℓ is given by:

$$H(\omega) = e^{-\gamma \ell} .$$

Then, from Equation 3.4 , it can be seen that at high frequencies a line's transfer function is

$$H(\ ) = e^{-(\sqrt{KCp\sqrt{p} + p^{2}LC})\ \ell} . \qquad \text{(eqn 3.47)}$$

Now $\sqrt{KCp\sqrt{p} + p^{2}LC}$ can be written as $p\sqrt{LC}\ \sqrt{1 + \frac{K}{L}\frac{1}{p}}$. For high frequencies, $\frac{K}{L}\frac{1}{\sqrt{p}}$ is very small so that $\sqrt{(1 + \frac{K}{L\sqrt{p}})}$ can be approximated using the rule $\sqrt{1+x} \approx 1 + (x/2)$ for $x<<1$. Thus, Equation 3.47 becomes

$$H(\omega) = e^{-p\sqrt{LC}\,\ell}\ e^{-\sqrt{p}\,\frac{K}{2}\sqrt{\frac{C}{L}}\,\ell} .$$

Substituting j for p, obtain

$$H(\omega) = e^{-j\omega\sqrt{LC}\,\ell}\ e^{-\sqrt{j\omega}\,\frac{K}{2}\sqrt{\frac{C}{L}}\,\ell} . \qquad \text{(eqn 3.48)}$$

This equation is analogous to Equation 3.39 so that the first term represents delay and the second represent attenuation [Ref. 44]. From the transfer function $H(\omega)$, it can be concluded the transmission line response to a high frequency signal is attenuation and delay of that signal.

2. Transmission Data Rate

Modern communications equipment has evolved from analog circuitry to very fast digital processors. As analog signals are replaced by pulse signals, the analysis of the pulse response of a transmission line is a topic of increasing importance. In the previous section, the high frequency line response (or transfer function) $H(\omega)$ was discussed and found to be:

$$H(\omega) = e^{-j\omega\sqrt{LC}} \quad e^{-\frac{K}{2}\sqrt{C/L}\sqrt{j\omega}\,\ell}$$

representing signal attenuation and delay. The former directly impacts transmission data rate and therefore is analyzed more closely.

The high frequency transmission line transfer function $H(\omega)$ can be viewed as consisting of two terms namely:

$$H(\omega) = H_1(\omega) \quad H_2(\omega)$$

where $H_1(\omega)$ represents the delay $e^{-j\omega\ LC}$ and $H_2(\omega)$ represents the attenuation $e^{-\frac{1}{2}K\sqrt{C/L}\sqrt{j\omega}\,\ell}$. To simplify the notation, let $t_0 = \ell\sqrt{CI}$, and define a new term ξ where:

$$\xi \triangleq \left(\frac{\ell\, K}{4\sqrt{L/C}}\right)^2 .$$

Also define another new term η where:

$$\eta \triangleq \frac{\ell\, K}{4\sqrt{L/C}\sqrt{\pi}}$$

so that $\sqrt{\xi} = \eta \sqrt{\pi}$. Therefore, the transfer function $H(\omega)$ can now be written as:

$$H(\omega) = e^{-j\omega t_0} e^{-2\sqrt{j\omega\xi}}. \qquad \text{(eqn 3.49)}$$

In order to obtain the transmission line data rate, the line response must be stated as a function of time rather than frequency. Thus the inverse Fourier transform of $H(\omega)$ must to determined. Since

$$H_2(\omega) = e^{-2\sqrt{j\omega\xi}},$$

Equation 3.49 becomes

$$H(\omega) = H_2(\omega)e^{-j\omega t_0}.$$

From basic properties of Fourier transforms, if $f(t)$ has Fourier transform $F(\omega)$, i.e. $f(t) \longleftrightarrow F(\omega)$ then

$$f(t-t_0) \longleftrightarrow F(\omega)e^{-j\omega t_0}$$

Therefore:

$$H(\omega) \longleftrightarrow h_2(t-t_0),$$

or

$$h(t) = h_2(t-t_0)$$

where $h_2(t) --- H_2(\omega)$. Through the use of Fourier transformation tables $h_2(t)$ is determined to be:

$$h_2(t) = \eta \, t^{-3/2} \, e^{-\xi/t} \, U(t)$$

where $U(t)$ is the unit step function depicted in Figure 3.11 . Then $h_2(t-t_0)$ is:

$$h_2(t-t_0) = \eta \, (t-t_0)^{-3/2} \, e^{-\xi/(t-t_0)} \, U(t-t_0)$$

which represents the function $h_2(t)$ delayed t_0 seconds.

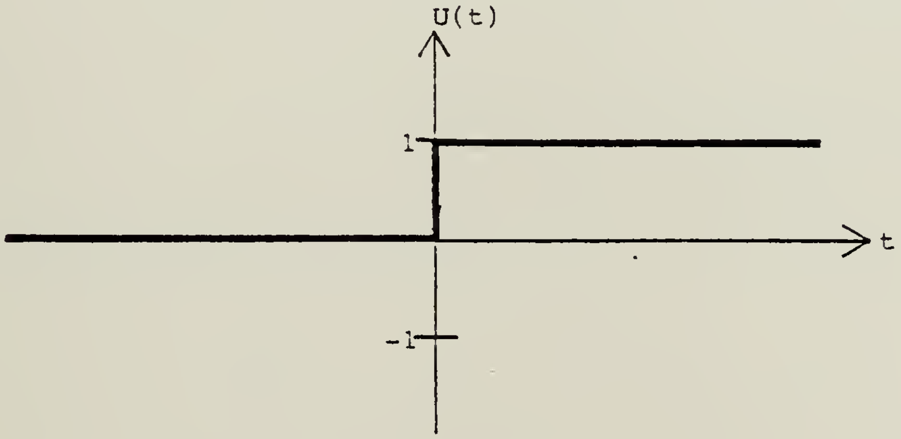


Figure 3.11 Unit Step Function U(t).

Example 3.4: If a coaxial cable 100 kilometers long has a line inductance L of 3.7×10^{-7} henries per meter and a line capacitance of 3.5×10^{-11} farads per meter, from the definition $t_0 = \ell\sqrt{LC}$ it can be determined that any high frequency signal applied to the line will experience a propagation delay of 360nanoseconds. In other words, the output response to a signal would lag the input by 360nanoseconds. The term $h_2(t)$ must now be analyzed.

Due to the presence of $U(t)$, $h_2(t)$ will be zero for $t \leq 0$. Furthermore, in the limit as t approaches infinity, $h_2(t)$ goes to zero. The value of $h_2(t)$, when $0<t<\infty$, must be examined. Since t is restricted to only positive values, by inspection, it can be seen that $h_2(t)$ will always be positive and be of the shape illustrated in Figure 3.12. The peak of the response curve is determined by setting the function's first derivative to zero. Then

$$\eta \left[t^{-7/2} \xi e^{-\xi/t} - \frac{3}{2} t^{-5/2} e^{-\xi/t} \right] = 0$$

which can be reduced to

$$t^{-1}\xi - \frac{3}{2} = 0$$

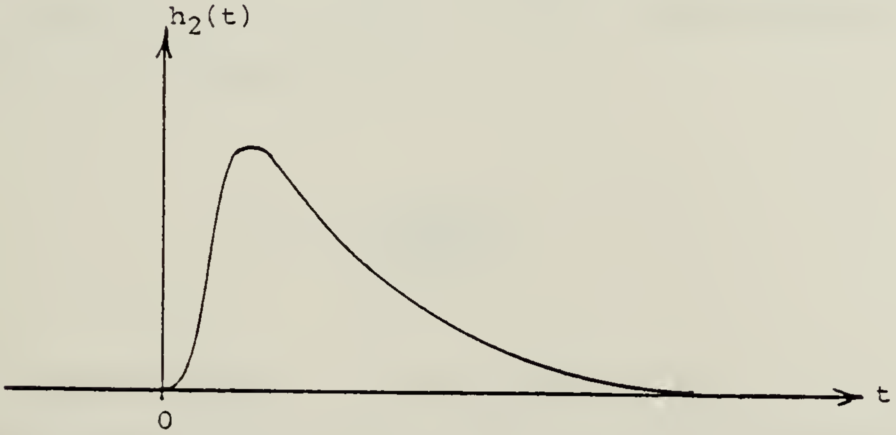


Figure 3.12 High Frequency Transmission Line Response Curve.

It can now be seen that the maximum value of $h_2(t)$ occurs at $t=(2/3)\xi$. The peak value can easily be derived by

solving for $h_2(\frac{2}{3}\xi)$. Thus the peak value of the response function is:

$$h\left(\frac{2}{3}\xi\right) = \frac{e^{-\xi/(\frac{2}{3}\xi)}}{\left(\frac{2}{3}\xi\right)^{3/2}}$$

$$= \eta\,\frac{e^{-3/2}}{\left(\frac{2}{3}\right)^{3/2}\xi\sqrt{\xi}} \quad .$$

Recalling that $\sqrt{\xi} = \eta\sqrt{\pi}$, then:

$$h_2\left(\frac{2}{3}\xi\right) = \frac{1}{\left(\frac{2}{3}e\right)^{3/2}\sqrt{\pi}\,\xi} \quad ,$$

Therefore, the peak value of $h_2(t)$ is approximately (0.23)(1/ξ).

Example 3.5: If the cable in the previous example has a skin effect resistance (K) of 1.17132x10^{-6} ohms per meter per second but the line is only 10kilometers in length, then ξ can be calculated from

$$\xi = \left(\frac{\ell K}{4\sqrt{L/C}}\right)^2$$

and is 0.8 nanoseconds. The peak value of the response function occurs when t=(2/3)ξ or at 0.54 nanoseconds and is h(0.54 nanoseconds) = 285x10^6.

It can be demonstrated that as ξ becomes smaller, the more closely the response curve resembles an impulse. With the aid of fourier transforms it can be shown that the transfer function of an impulse is unity for all frequencies. Therefore, it can be concluded that the smaller the ξ

of a transmission line, the better suited the system is for high frequency signal transmission. So, for the ideal transmission line, $H(\omega)=1$. However, as has been demonstrated, the response of a real transmission line is not ideal as indicated by the fact that

$$H(\omega) = e^{-2\sqrt{j\omega\xi}}.$$

Clearly $|H_2(\omega)| < 1$ for all frequencies. $H_2(\omega)$ can be written in polar form, that is:

$$H_2(\omega) = \left|H_2(\omega)\right| e^{j\angle H_2(\omega)} .$$

Since

$$H_2(\omega) = e^{-2\sqrt{\omega\xi}}\, e^{\sqrt{j}}$$

and

$$\sqrt{j} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} j ,$$

then

$$H_2(\omega) = e^{-\sqrt{2\omega\xi}}\, e^{-j\sqrt{2\omega\xi}} .$$

Clearly

$$\left|H_2(\omega)\right| = e^{-\sqrt{2\omega\xi}}$$

and

$$\angle H_2(\omega) = -\sqrt{2\omega\xi}.$$

Figure 3.13 illustrates the frequency response of the transmission line as determined from $|H_2(\omega)|$. It can be seen that the line has the frequency response characteristics of a low pass filter. The half power point of $|H_2(\ )|$ occurs at the frequency ω such that $\omega\xi = 0.06$. Therefore, it is

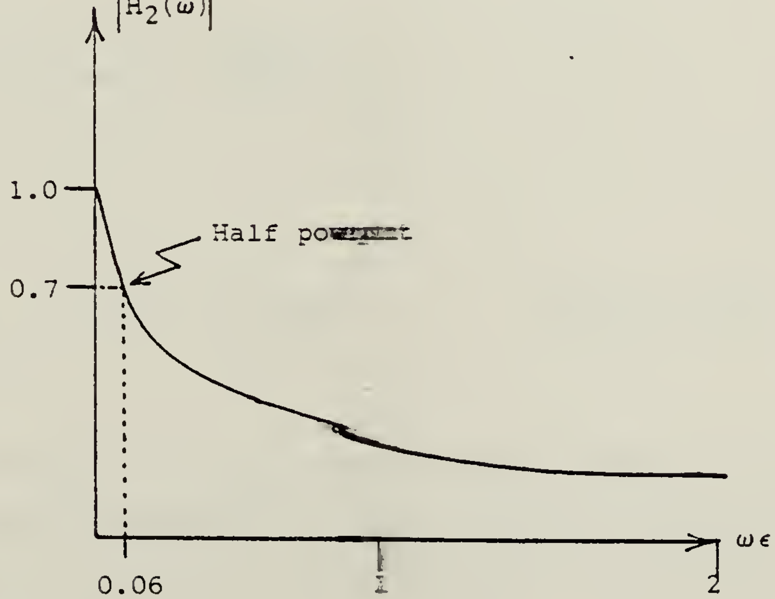


Figure 3.13 Average Power Spectral Density.

clearly demonstrated that the ξ value of the line determines the cut-off frequency of the line.

It is often useful to specify frequency response of a line in the context of dB power loss at various frequencies. It can be demonstrated that the output spectral density of a linear system is related to the input spectral density to the system by [Ref. 45]

$$P_{out}(\omega) = P_{in}(\omega)\ |H_2(\omega)|^2$$

where $P_{out}(\omega)$ represents output power spectral density and $P_{in}(\omega)$ represents input power spectral density. Since

$$\frac{P_{out}(\omega)}{P_{in}(\omega)} = |H_2(\omega)|^2$$

The power loss (in dB) for any given fixed frequency is given by

$$(\text{Loss})_{dB} = 10\ \log(e^{-\sqrt{2\omega\xi}})^2$$

$$(\text{Loss})_{dB} = -12.28\sqrt{\omega\xi}. \qquad \text{(eqn 3.50)}$$

It can therefore be concluded that any transmission line system which does not "condition" the line with equalizers is limited in its frequency response by the line's ξ value. Realizing that modern telecommunication systems are exclusively digital systems, since digital signals are typically high frequency signals, it is apparent that the transmission line parameters determine the maximum number of pulses per

second that can be transmitted over a line. Therefore it is important to determine the line response to a pulse.

Lathi [Ref. 46] has shown that a unit step response U(t) can be expressed as the integral of a unit impulse function $\delta(t)$.

$$U(t) = \int_{-\infty}^{t} \delta(\tau)d\tau \quad .$$

The unit step response, $r_2(t)$, of the line is given by:

$$r_2(t) = \int_{\infty}^{t} h_2(\tau)d\tau \quad .$$

This follows from the previous equation and linear system theory. Substituting for $h_2(\tau)$ yields

$$r_2(t) = \int^{t} \eta \frac{e^{-\xi/\tau}}{(\tau)^{3/2}} U(\tau)\, d\tau \quad .$$

Since

$$U(\tau) = \begin{cases} 0 & \text{for } \tau < 0 \\ 1 & \text{for } \tau \geq 0 \end{cases}$$

then

$$r_2(t) = \left[\int_{0}^{t} \frac{e^{-\xi/\tau}}{(\tau)^{3/2}}\, d\tau\right] U(\tau) \quad .$$

Letting x be a variable of integration such that $x = \sqrt{2\ /}$ it can be demonstrated that the line's unit step response is given by

$$r_2(t) = 2\int_{\sqrt{\frac{2\varepsilon}{\tau}}}^{\infty} \frac{1}{\sqrt{2\pi}}\, e^{-(\frac{x^2}{2})} dx \ . \qquad \text{(eqn 3.51)}$$

The intergal of Equation 3.51 is the so-called complementary error function (CERF) of a normal distribution, where

$$\int_x^{\infty} \frac{1}{\sqrt{2\,\pi}}\, e^{-(\frac{x^2}{2})} dx = 1 - F(x) \qquad \text{(eqn 3.52)}$$

and F(x) is the so-called error function integral [Ref. 47]. From Equations 3.51 and 3.52 the unit step response of a transmission line can be determined from

$$r_2(t) = 2[1 - F(\sqrt{2\xi/\tau})] \qquad \text{(eqn 3.53)}$$

and is illustrated in Figure 3.14 .

Figure 3.14 represents the signal $r_2(t)$ at the output terminals of the transmission line in response to a unit step input signal. The time required for the output level to go from 10% to 90% of the maximum output signal value is called the rise time. Letting (t_i) denote the time at which the output reaches the 10% signal level and (t_f) denote the time at which the output reaches the 90% level, rise time is $t_f - t_i$. Therefore combining Equation 3.53 and its graphic representation in Figure 3.14 , it can be seen that

$$r_2(t_i) = 0.1 \quad \text{and} \quad r_2(t_f) = 0.9 \ .$$

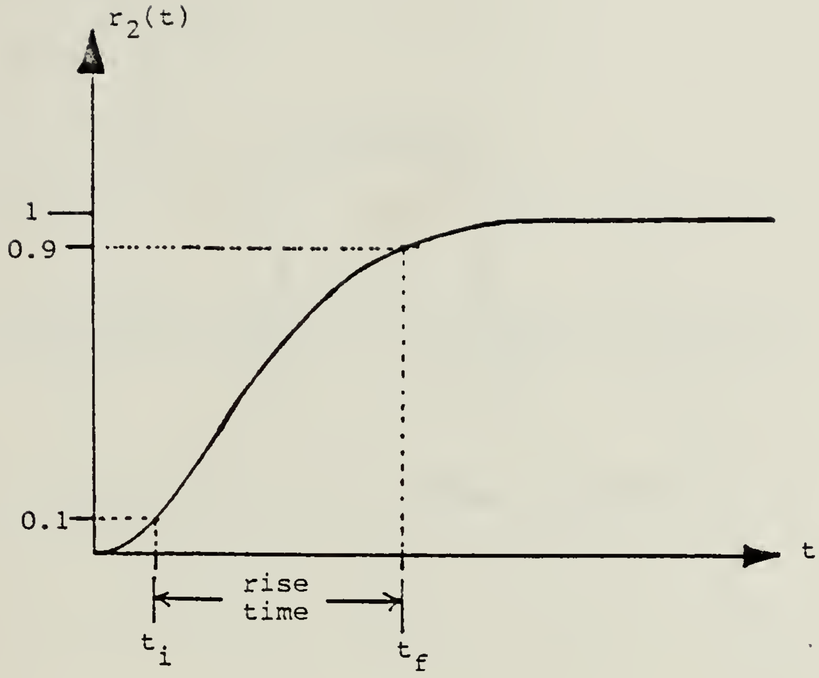


Figure 3.14 Unit Step Response.

It follows then that through substitution of t_i and t_f into Equation 3.53 that $t_i = 0.74$ and $t_f = 128$. Therefore rise time, denoted t_r, associated with a transmission line is 127.26ξ seconds. For example if a transmission line has a value of $\xi = 8.111 \times 10^{-10}$ seconds, then the rise time of the response due to a unit step signal applied at the input of the line would be 100nanoseconds.

Now that the transmission line response to a unit step signal is known, the line response to a pulse can be easily determined since a pulse can be viewed as two unit step signals occurring at different times. Let the pulse signal of Figure 3.15 be represented as U(x) - U(x-d). Figure 3.16 represents the individual responses of U(x) and U(x-d) such that

$$r_2(x) = 2[1 - F(\sqrt{2/x}\,)]\ U(x)$$

and

$$r_2(x-d) = 2[1 - F(\sqrt{2/(x-d)}\,)]\ U(x-d)\ .$$

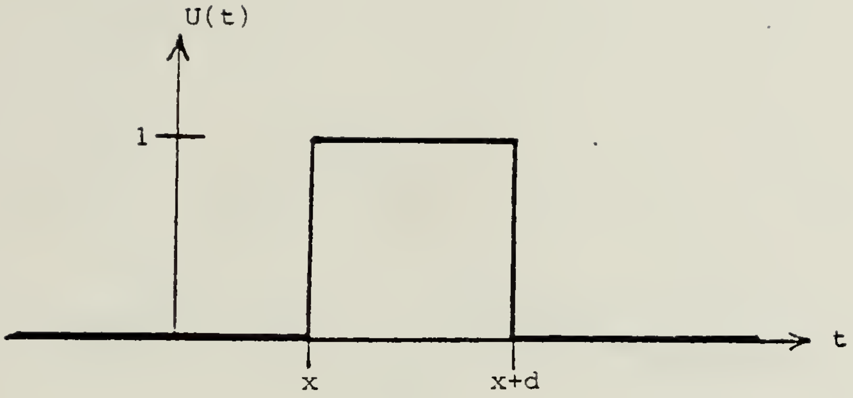


Figure 3.15 Pulse Signal.

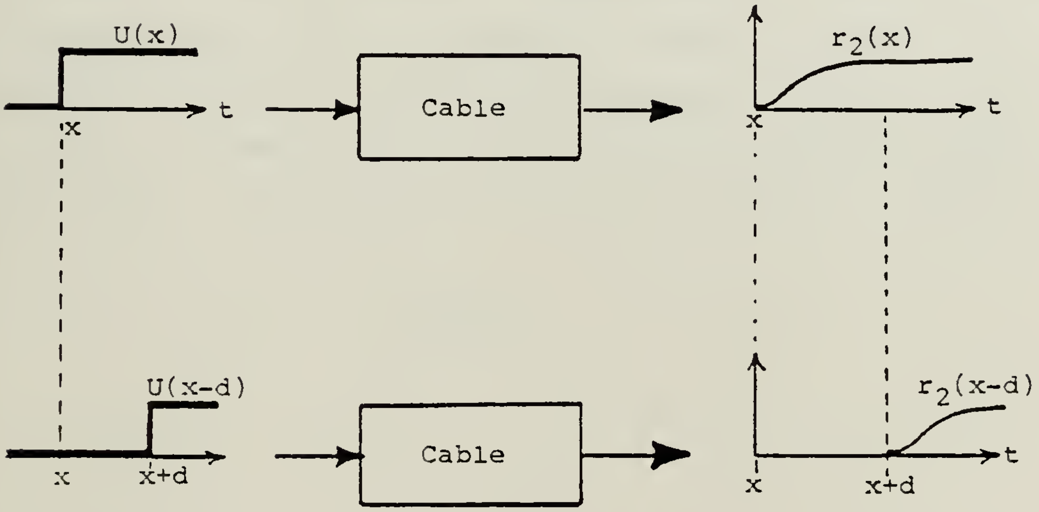


Figure 3.16 Individual Line Responses.

From $U(x)-U(x-d)$, the resultant response is $r_2(x)-r_2(x-d)$ as illustrated in Figure 3.17 .

It can be seen from Figure 3.17 that as the pulse duration d is made smaller the response pulse amplitude decreases. Most systems base cutoff signals at the 3dB signal power level. Therefore, d should be long enough to insure that the voltage (or current) response pulse is at

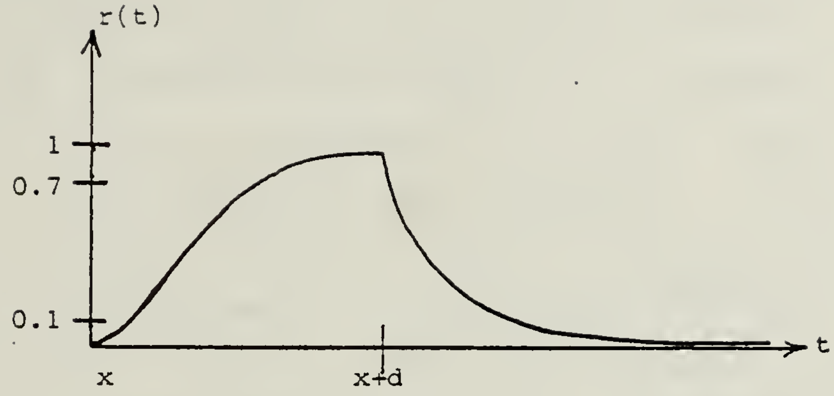


Figure 3.17 Line Response to a Pulse.

least 70.7% of the input signal voltage (or current) level. To maximize the number of pulses put on a line the pulse width d should be minimized. Therefore d should be based on the time required for a response pulse to rise to at least the half power point of the input signal power. Thus, $r_2(x)$ should be set to reach the 0.707 voltage (or current) level. The pulse width is found by setting Equation 3.53 to 0.707 so that

$$2[1 - F(\sqrt{2\xi/\tau})] = 0.707.$$

Letting $\sqrt{2\xi/\tau} = x$, then $F(x) = 0.646$. The value of $F(x)$ can be found from normal distribution tables so that $x=0.37$. Therefore, it can be seen that

$$\sqrt{2\xi/\tau} = 0.37$$

so that $t = 14.6\,\xi$. For simplicity, the pulse duration is choosen as multiples of the lines ξ value so that the cut off pulse will occur $15\,\xi$ after the turn on pulse. That is, pulse duration is $15\,\xi$ seconds. Likewise a reasonable

amount of delay is required to ensure the line is at a steady state prior to turn on of another pulse. By the same technique used to develop the width of a pulse, the interval between the turn off pulse and the new turn on pulse is found to be approximately 15ξ seconds. The total time between the leading edge of the two consecutive pulses is approximately 30ξ seconds. Therefore, the data rate of any given transmission line is $1/30\xi$ and clearly depends on the characteristic parameters of the line as related by ξ .

Example 3.6: If a transmission line has an ξ value of 8.111×10^{-10} seconds, then the optimal pulse width of a pulse applied on the line is 12nanoseconds. The maximum data transmission rate is approximately 41Mbps (Mega bits per second).

3. Signal-to-Noise Ratio

The previous section briefly discussed the power loss experienced by a signal of a certain frequency as it propagates through the line. From Equation 3.50 this power loss is given by

$$\text{dB Loss} = -12.28\sqrt{\omega\xi}$$

or

$$\text{Loss} = 10^{-1.228\sqrt{\omega\xi}}.$$

From earlier chapters of this thesis it was demonstrated that for a lossy device the noise figure F is 1/Loss. Thus, the noise figure is given by

$$F = 10^{1.228\sqrt{\omega\xi}}.$$

However, since noise figure is usually specified in dB, a transmission line's noise figure is

$$(F)_{dB} = 12.28\sqrt{\omega\xi} \ .$$

Clearly, a transmission line's noise figure is dependent on both the line's ξ value and its operating frequency. Furthermore, it has be demonstrated that ξ is a function of both a line's characteristic parameters and its length. Thus it can be determined from the SNR equation (Equation 2.74) that for a transmission line

$$\mathrm{SNR} = = \frac{\text{Power input}}{(16.9\times10^{\sqrt{\omega\xi}} - 1)kT\ B} \ .$$

It can therefore be concluded that as frequency or the length of any given cable increases, system performance is degraded by virtue of the decreasing SNR.

IV. WAVEGUIDES

Although the basic physical theory of waveguides has been known since the nineteenth century, waveguide theory and technology were not developed until World War II when the design of microwave radar generated the need for practical high frequency transmission systems [Ref. 48]. It was demonstrated in Chapter 3 that the attenuation constant of a transmission line is proportional to the square root of the line frequency (Equation 3.50). Thus as system operating frequencies increase so too do the line losses. The development of the waveguide provides a viable transmission channel which exhibits lower loss at the higher frequencies than cable systems. As the demand for communication channels increased, the communications industry viewed microwave transmission as a profitable alternative to increasing the land line network and adopted waveguide technology from the original radar applications. Today, waveguides have become an integral part of the world-wide telecommunication structure. They have provided the means to develop vast microwave systems and the ever-expanding space communications capabilites which significantly contribute to the lowering of consumer commercial and military long distance communication costs. Additionally, through microwave and satellite systems, entire nations typified by a collection of isolated regions such as Indonesia are able to install affordable nation-wide communication networks.

Like antenna and transmission line systems, waveguides represent a means of transmitting electronic signals from one point to one or more other points. While waveguides are usually treated separately from transmission lines, they are in many ways identical. Accompanying any voltage and

current in a line are electric and magnetic fields. The behavior of the transmission line can be analyzed in terms of the accompanying electromagnetic fields between and around the wires instead of in terms of voltages and current. However, the voltage-current analysis is carried out here as it is more understandable than the field analysis. A strict analysis of waveguides can only be carried out in terms of the electromagnetic fields; but because of the voltage-current and electromagnetic field relationships, analogies to the transmission line equations can be used that both simplify the analysis and increase the understanding of the behavior of waveguides.

Chapter 2 presented an analysis of antenna systems and demonstrated that, while antennas transmit their energy in a preferred direction, the propagated signal is (generally) not confined to a closed path. As a result, the signal propagates as an ever-expanding wave such that the signal field density decreases at a distance squared rate. Waveguides, which are nothing more than hollow metal tubes, provide a closed path through which the transmitted signal propagates as a non-expanding plane wave. Thus, if the waveguide were lossless, the field density of the plane wave at the output end of the waveguide would equal the field density at the input side of the waveguide. However, waveguides are not lossless and the input signal energy tends to dissipate along the walls of the guide (or absorbed by other means) as the plane wave propagates through the guide. Because analysis of a waveguide is based on electromagnetic wave propagation, a brief discussion of wave propagation is required. The cross-sectional shape of waveguides can be rectangular, circular, or other more complex designs. Since, however, the rectangular waveguide is by far the one most commonly used, further discussions will be limited to this shape. [Ref. 49].

A. WAVES IN GUIDES

Electromagnetic (EM) fields are characterized by a magnitude and direction in space so that they are vector quantities. Furthermore, an EM field is a combination of an electric field and a magnetic field such that each point in space has associated with it an electric field vector, E, and a magetic field vector, H. When EM waves propagate in vacuum the E and H vectors are always at right angles to each other at any instant in time, and they are at right angles to the direction of propagation of the wave. A wave radiated from a point source (an antenna) produces such a wave which radiates as an expanding sphere. In a waveguide, the signal energy is confined to the space within the guide walls so that rather than propagate as an expanding spherical wave, the signal propagates down the guide as a plane wave. The fields in a waveguide have similar general properties as fields in free space, but, because of the confinement of the fields by the waveguide walls, there are some differences [Ref. 50].

Confinement of the EM wave prevents spherical spreading. In free space, where waves spread spherically, the E and H vectors are always parallel to the direction of wave propagation. Within waveguides two or more waves are reflected back and forth which produces a propagating wave having compcntents of either E field or H field vectors, but never both. Maxwell's equations are the basis for determining resultant waves propagated in the waveguide and, because they are partial differental equations, there can be more than one solution; that is, more than one configuration of the propagating EM field is possible. These configurations, called modes, may exist separately or several may exist simultaneously. Ordinarily it is desired to operate the waveguide with only one of the possible modes of propagation. [Ref. 51].

The modes of propagation that a waveguide can support are divided into two classes: (1) transverse electric (TE) modes and (2) transverse magnetic (TM) modes. In the TE modes, the E field is everywhere perpendicular to the direction of propagation. This means there is no E field component parallel to the guide axis. In the TM mode the same is true for the H field component. Each mode of propagation is designated TE_{mn} or TM_{mn} where m and n are integers and are called mode indices. The mode indices are functions of the relationship between waveguide dimensions and transmitted signal wavelength. Each operating mode has associated with it a cutoff frequency. As the mode index (m,n) increases so too does the cutoff frequency. For example, the cutoff frequency for TE_{23} is higher than the cutoff frequency for TE_{11}. For various engineering reasons, the TE_{10} mode is the preferred mode of the rectangular waveguide and yields the lowest cutoff frequency for that guide design. [Ref. 52].

1. Cutoff Frequency

Figure 4.1 is a diagram of a section of rectangular waveguide. It is customary to denote the larger of the two

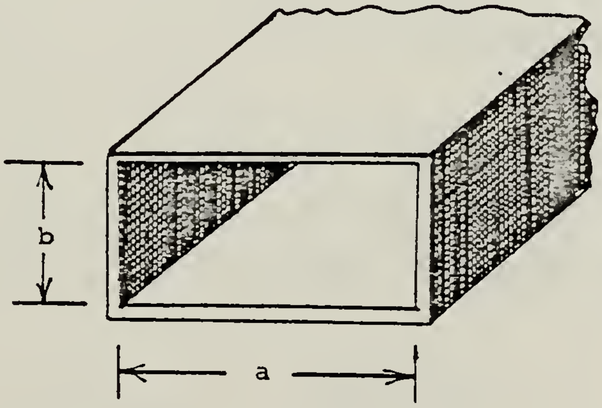


Figure 4.1 Rectangular Waveguide.

dimensions by a, and the smaller by b. The EM field configuration corresponding to a particular mode that can propagate in the waveguide must have a frequency which is greater than the cutoff frequency, f_c, for that particular mode. The cutoff frequency for a rectangular guide can be expressed mathematically as a function of the guide dimensions a and b, the mode indices m and n, and the permeability μ and permittivity ϵ of the medium enclosed by the waveguide walls. In terms of these quantities, the cutoff frequency is given by

$$f_{c\,(m,n)} = \frac{c}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} \qquad \text{(eqn 4.1)}$$

where c is the velocity of light in vacuum. It can be shown from this equation that as the values of the mode indices decrease so too does the cutoff frequency. Conversely as the size of the guide decreases, the cutoff frequency for any given mode increases. It must be noted that in either the TE or TM mode for any waveguide the indices m and n can never both be zero. Then, given a > b it becomes readily apparent that, as previously stated, the lowest cutoff frequency for the rectangular waveguide is associated with the TE_{10} mode. Figure 4.2 shows the relationship of various mode cutoff frequencies for two values of the ratio a/b [Ref. 53].

Example 4.1: A typical air filled waveguide ($\epsilon \simeq 1$ and $\mu \simeq 1$) has dimensions a = 8.6 centimeters and b = 4.3 centimeters. The TE_{10} mode cutoff frequency is

$$f_{c\,(1,0)} = \frac{3\text{x}10^8\ \text{cm/sec}}{2}\sqrt{\left(\frac{1}{8.6\text{cm}}\right)^2} = 1.744\text{GHz}\ .$$

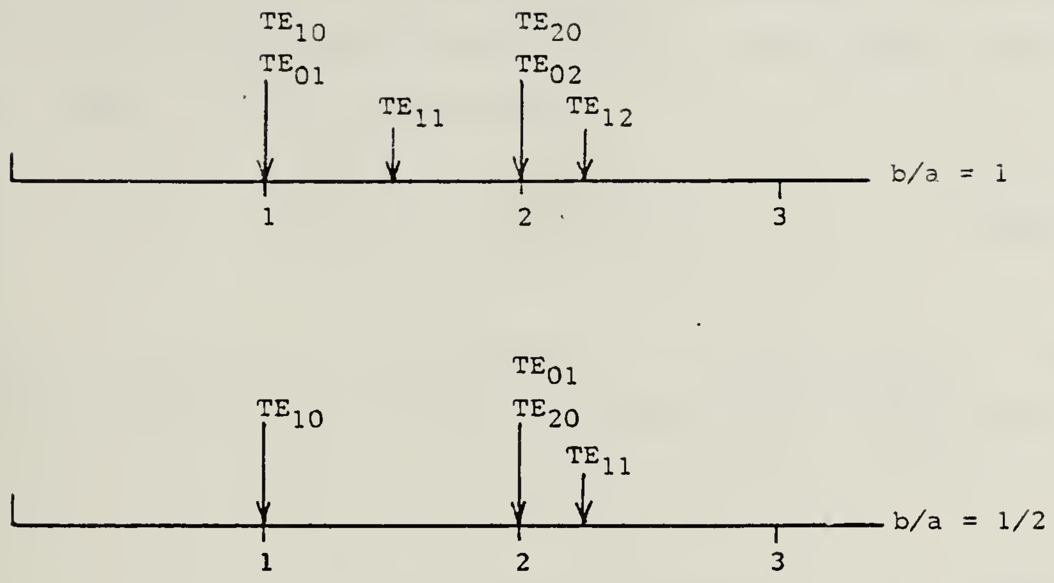


Figure 4.2 Relative Cutoff Frequencies.

The TE_{01} mode cutoff frequency is 3.488 GHz, which is twice the TE_{10} mode cutoff.

2. Wave Velocity and Guide Wavelength

In free space or any nearly lossless propagation medium whose dimensions are very large compared to the transmission signal wavelength, the wave velocity is determined only by the permeability and permittivity of the propagation medium. That is, for a vacuum μ and ϵ are unity, so that wave velocity for any frequency in free space is c. In hollow-pipe waveguides, however, the wave velocity does, in general, vary with the frequency, even with an air filled or evacuated guide. Moreover, it becomes necessary to distinguish between two concepts of velocity - the phase velocity and the group velocity of waves. [Ref. 54].

The phase velocity V_{ph} is the velocity with which the phase of the wave advances through the medium. The group velocity V_{gr} is the velocity with which energy (or eventually information) is propagated by a wave. In free space and on lossless transmission lines V_{ph} and V_{gr} are equal (or low frequency lossy transmission lines V_{ph} and V_{gr}

are nearly equal); therefore the distinction between them was not made. In waveguides, the two velocities are related to each other by the expression

$$(V_{ph})(V_{gr}) = c^2/(\mu\epsilon) \quad \text{(eqn 4.2)}$$

The phase velocity in waveguides is expressed in terms of the cutoff frequency f_c and the operating frequency f such that:

$$V_{ph} = \frac{c}{\sqrt{\mu\epsilon}\ \sqrt{1-(\frac{f_c}{f})^2}} \quad . \quad \text{(eqn 4.3)}$$

Thus from Equation 4.2 and 4.3 the following expression for V_{gr} is obtained:

$$V_{gr} = \frac{c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}{\sqrt{\mu\epsilon}} \quad . \quad \text{(eqn 4.4)}$$

Since f must always be equal to or greater than f_c if a wave is to propagate, it is evident that V_{ph} is never less than $c/\sqrt{\mu\epsilon}$ and V_{gr} is never greater than $c/\sqrt{\mu\epsilon}$. Therefore it can be seen that V_{ph} is always equal to or greater than V_{gr} . It can then be shown that the wavelength λ_g of a given propagating frequency, is greater than the wavelength λ_f of the same frequency propagating in free space. Therefore the guide wavelength is determined by

$$\lambda_g = \frac{c}{\sqrt{\mu\epsilon}\ \sqrt{f^2-f_c^2}} \quad . \quad \text{(eqn 4.5)}$$

This value λ_g is used for all dimensioning and measurement in waveguides [Ref. 55].

B. GUIDE OPERATING CHARACTERISTICS

The transmission line equations of chapter 4 are in terms of currents, voltages, and impedances. The quantities in waveguides analogous to current and voltage in these equations are the H and E fields respectively.

1. Characteristic Impedance

Since impedance in circuits is defined as the voltage to current ratio, it follows that the analogous quantity in waveguides is the E/H ratio. In fact, this ratio is called the wave impedance. Furthermore, just as transmission lines have a characteristic impedance, waveguides have a characteristic wave inpedance. As can be demonstrated by a careful analysis of waveguides, the characteristic wave impedance for the TE_{10} mode is given by:

$$Z_{o(TE)} = \frac{1}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

where η is the intrinsic or characteristic impedance of the medium enclosed by the waveguide. For an air filled or evacuated guide $\eta = \sqrt{\mu_0 / \epsilon_0}$ where μ_0 is the permeability (1.257×10^{-6} henries per meter) and ϵ_0 is the permittivity (8.854×10^{-12} farads per meter) of vacuum. Thus for the air filled or evacuated waveguide, its characteristic wave impedance is

$$Z_{o(TE)} = \frac{120}{\sqrt{1-(f_c/f)^2}} \qquad \text{(eqn 4.6)}$$

It can be seen that at frequencies well above the cutoff frequency the characteristic wave impedance approaches 120π. [Ref. 56].

2. Propagation Constant

From transmission line analogies it should become evident that waveguides have associated with them a propagation constant. The propagation constant of a waveguide is given by the same general equation as the transmission line propagation constant, namely:

$$\gamma = \psi + j\beta$$

where ψ is related to attenuation and β is related to phase delay. As already discussed, two velocity factors (V_{ph} and V_{gr}) are associated with waveguides. The phase velocity V_{ph} is important in determining waveguide dimensions as previously pointed out. The group velocity (V_{gr}) has an interesting interpretation as the velocity of energy or information flow in the waveguide system so that the velocity of concern regarding propagation of data is V_{gr} and is determined from

$$V_{gr} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{1-(f_c/f)^2} \qquad \text{(eqn 4.7)}$$

For an air filled or evacuated guide $V_{gr} = c\sqrt{1-(f_c/f)^2}$. In conjunction with this, the phase constant β is related to V_{gr} by the operating frequency of the guide such that

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-(f_c/f)^2} \quad . \qquad \text{(eqn 4.8)}$$

For air filled and evacuated guides the phase constant is given as $=2\pi c\ \sqrt{f^2-f_c^2}$. [Ref. 57].

Initial assumptions for waveguide analysis include a lossless dielectric and perfectly conducting walls. Under these conditions an expression for the propagation constant γ is normally derived. Evaluating γ at frequencies below cutoff yields a purely real number which represents attenuation. At any frequency above cutoff γ becomes purely imaginary. However, since no guide is perfect, losses do exist for frequencies above cutoff. Formulas for attenuation factors for waveguides can be found in many textbooks and handbooks. While $\psi \doteq \mathrm{Re}\{\gamma\}$ (recall $\gamma = \psi + j\beta$) is extremely complicated for waveguide applications, the attenuation constant is readily workable. For the dominant TE_{10} mode in rectangular guides the result is

$$\alpha = \frac{8.7R_s}{b\eta K} \quad 1 + \left[\frac{2bf_c^2}{af^2}\right] \quad \text{decibels per meter}$$

where R_s is than surface resistance or the waveguide walls; $R = \sqrt{\pi f \mu_m/\sigma_m}$, $\eta = \sqrt{\mu/\epsilon}$, $K = \sqrt{1-(f_c/f)^2}$, and μ_m and σ_m are the values of permittivity and permeability of the the waveguide wall materials [Ref. 58].

For an air filled guide the attenuation constant simplifies to

$$= \frac{4.1\text{x}10^{-2}\sqrt{f\mu_m/\sigma_m}}{b\sqrt{1-(f_c/f)^2}} \left[1 + \frac{2bf_c^2}{af^2}\right].$$

It can be shown that at the cutoff frequency ($f = f_c$) the denominator is zero so that attenuation is infinite. Conversely, as $f \longrightarrow \infty$ the attenuation constant again becomes ∞. Therefore, there is an optimum frequency which yields the smallest loss. Usually, the operating frequency

is chosen such that it is between the TE_{10} and TE_{01} cutoff points.

C. WAVEGUIDE SNR

Like all other transmission systems, waveguide systems are evaluated in terms of their output signal to noise ratio. The overall noise figure F of a wave guide is found in the same manner as is done for transmission lines and is equal to the inverse of the system loss. Therefore the noise figure of a waveguide of length ℓ and an attenuation constant of α is

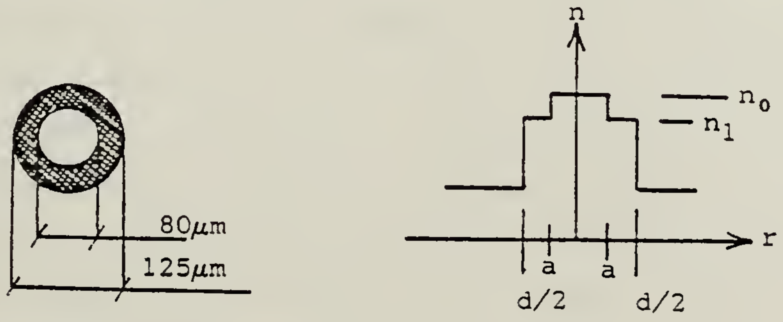
$$F = \frac{1}{10^{-0.1\alpha\ell}} = 10^{-0.1\alpha\ell}.$$

The general equation for a system's output signal-to-noise ratio is used so that for the waveguide just described, $SNR = 1/[(10^{-0.1\alpha\ell} - 1) kT_0 B]$. It can then be seen that the quality of signal transmission within a waveguide system is a function of the signal frequency, waveguide dimensions, dielectric characteristics of both waveguide wall materials and the enclosed medium material, and the length of the guide. To gain more favorable SNR values over a given guide distance the internal walls of the guide can be coated with a material having a low surface resistance such as silver or gold, replacing the existing dielectric within the waveguide with one having a lower intrinsic impedance, or both.

An alternate solution which is at the forefront of technology is to increase the carrier frequency to the visible light region of the electromagnetic spectrum. The practical use of this technology is increasing at a rapid rate. Therefore, fibers operating in the visible light region of the EM spectrum are considered next.

D. THE OPTICAL FIBER

Optical fibers are dielectric waveguide structures that are used to confine and guide electromagnetic waves whose frequencies are in the visible light spectrum. Usually discussions regarding the visible light spectrum focus on light wavelength rather than frequency. For visible light the wavelengths are between 0.5 and 1.5 microns(m). An optical fiber consists essentially of an inner dielectric material, called the core, surrounded by another dielectric with a smaller refractive index and is referred to as the cladding. All optical fibers currently in use have a circular cross-sectional shape and are either glass or fused silica. Plastics could be used but, in general, would have much higher losses due to impurities inherent in the material. While a variety of waveguide configurations can be manufactured, the waveguide configuration depicted in Figure 4.3 has been the one adopted as the most practical. The fiber core, having a refractive index n_0, is made of a material which exhibits low attenuation in the visible light spectrum. A cladding, which has a refractive index n_1 that is less than that of the core's index, surrounds the light transmitting core material [Ref. 59].



a) Dimensions b) Refractive Index

Figure 4.3 Perferred FO Configuration.

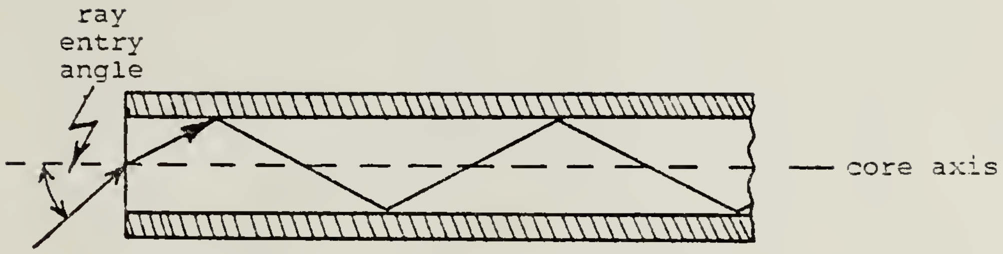


Figure 4.4 Light Ray Input Angle.

The most important parameters for specifying optical fiber waveguide properties are: core radius, (a), the numerical aperture (NA) defined as $NA = \sqrt{n_0^2 - n_1^2}$ and is related to the maximum acceptable off axis angle for rays entering the fiber (see Figure 4.4), and a characteristic parameter V defined as $V = (2\pi\ a/\lambda)\sqrt{n_0^2 - n_1^2}$ where λ is the transmitted light wavelenght in a vacuum. The step index structure depicted in Figure 4.3 results in multiple propagation modes which is found to equal $V^2/2$ and therefore is proportional to the core radius squared and the numerical aperture squared. As each mode travels through the waveguide with a different internal velocity, a high number of modes causes large signal distortion and therefore limits usable bandwidth. Thus in the context of telecommunication systems applications, the step index fiber is of limited practical use. However, recent manufacturing techniques have allowed the refractive index of the fiber core material to vary from the center of the core to the cladding boundary region in a specified fashion. The pictoral representation of this varying index, called a graded index, in Figure 4.5 illustrates this. The physics of the graded index fiber are such that all modes have nearly the same group velocity so that the differential mode delay, which tend to decrease fiber bandwidth, is thereby greatly reduced. Thus for lone-distance, high data rate applications, the graded index optical fiber is preferrable. [Ref. 60].

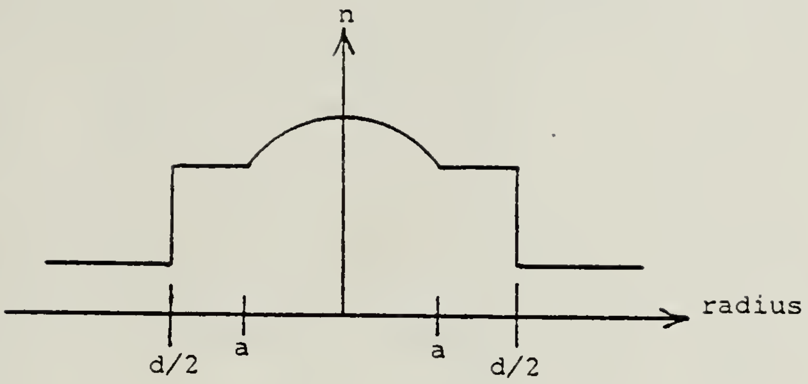


Figure 4.5 Graded Index FO.

The transmission properties which are of chief interest for telecommunication use are attenuation and dispersion of the input signal. A brief discussion of these factors is now presented.

1. Attenuation

Attenuation in optical fiber waveguides has historically been the benchmark of performance. The power losses in fiber optic lines are caused by material absorption and scattering, waveguide scattering, and radiation losses. Material absorption and scattering is mainly caused by ions within the glass itself. Waveguide scattering is caused mainly by irregularities at the core-cladding interface. Quality control at the manufacturing facilities can limit waveguide scattering losses to less than 1 dB/Km. Radiation losses are caused by the bending of a fiber in a small radius of curvature. Radiation losses are particularly high when cabling is installed without plastic cushioning material surrounding it [Ref. 61].

To minimize intrinsic material losses the fiber is fabricated in a manner so as to obtain a low concentration of impurities and to match the dielectric properties of all compontents within the glass in order to minimize scattering from compositional fluctuations [Ref. 62].

Macro-bending of optical waveguides causes radiation losses which become increasingly severe with decreasing bend radius. The smallest permissible curvature radius is limited by the actual fiber strength. Different fibers and cables have different strain-to-failure properties depending upon construction methods and materials. If a fiber is bent around a surface of radius R, known as the bend radius, the outermost edge of the fiber cladding of radius r will be strained relative to the fiber axis by a determinable percentage σ_s, where $_s = \left[\left(\frac{R+2r}{R+1}\right) - 1\right]$ x 100%. For fibers and cables to maintain longevity, the maximum differential bending strain (σ_s) should not exceed 0.2% [Ref. 63].

Due to all the variables it is impractical to attempt development of a generalized power loss equation for optical fiber waveguides. However, as discussed in the opening section of this chapter for the generic waveguide, signal attenuation in the guide is a function of frequency. From empirical data it has been concluded that there are two attenuation minima for fiber waveguides. These occur between 0.8 and 0.9 microns and at 1.05 microns which are usually referred to as first and second windows respectively. Currently, attenuation values for optical fibers range between 3 and 5 dB/Km for the first window and 1 dB/Km for the second [Ref. 64].

2. Pulse Dispersion

Pulse dispersion is a distortion corresponding to a band limitation of the fiber and is caused by modal dispersion and material dispersion. Modal dispersion is due to the difference of group velocity of the different modes propagating along the fiber at a single wavelength. This effect can be reduced in multimode fibers through a graded refractive index of the core. In practical low-loss fibers normal values of pulse broadening (τ_M) as low as 1 nanosecond per kilometer are specified [Ref. 65].

Material dispersion is associated with the bandwidth of the optical signaling source and is caused by the variation of the refractive index with the optical wavelength. With light emitting diode (LED) sources material dispersion is the predominant effect with pulse broadening values in the 2-5 nsec/Km range. If a laser source is used, the pulse broadening values can be substantially lowered [Ref. 66].

Since pulse broadening occurs in the fiber as the light signal propagates down the guide, an optical fiber waveguide bandwidth is limited by its pulse broadening value. The fiber system which has an LED source has a bandwidth which varies between 10 and 100 MHz. The system can be improved by using a laser source which increase the guide bandwidth to the 500 - 1000 MHz range [Ref. 67].

V. SUMMARY AND CONCLUSIONS

The central theme of this thesis has been the transmission of information in the form of electrical energy. The various forms of energy transmission have been grouped according to the number of conductors involved and include antennas, waveguides, and transmission lines. Among the tasks of the telecommunication systems manager is to choose the type of transmission channel which will best satisfy user/system constraints and requirements. The mode chosen for any one particular application depends on a number of factors, such as: (1) life cycle cost; (2) frequency band and information-carrying capacity; (3) selectivity or privacy offered; (4) reliability and noise characteristics; and (5) power level and efficiency. Each of the transmission channels has only some of the desirable features; consequently, the selection of the channel best suited for a specific application is a matter of managerial as well as engineering judgement. The underlying principles which determine the general characteristics of each transmission channel have been presented. It is these principles which managers must understand to insure their decisions do not adversely impact the user's mission and the system's intended capabilities.

In order to emphasize why it is necessary to understand the characteristics of the communication channel, consider the problem of transmitting an ultra-high frequency (UHF) information signal at 1000 MHz between two points 30 miles apart. Without knowledge of the various channel characteristics, one could erroneously choose to use either the waveguide or transmission line as an efficient means of transmitting the signal. However, for this particular case,

the antenna will be superior to either the transmission line or the waveguide. If the received signal power in each case were chosen as the system's figure of merit, then for comparison it would be found that for reasonably typical installations, the transmitted power required to produce 1 nanowatt at the receiver's input would be on the order of:

1. Transmission lines: 10^{545} watts (5540dB loss).
2. Waveguides: 10^{18} watts (270dB loss).
3. Antennas: 50 milliwatts (77dB loss).

These figures were derived from an assumed transmission line loss of 3.5dB/100 ft (RG-19A/U cable loss obtained from Figure 5.1), an assumed waveguide loss of 0.17dB/100 ft (rectangular guide loss obtained from Figure 5.2), and transmitting and receiving antennas with an effective area of 2 square meters (effective antenna gain was obtained from Equation 2.23 and found to be 24.5dB each). Thus the most power efficient system over this 30 mile distance is the one using antennas, i.e. the free space channel.

These results can be quickly obtained from the principles presented in Chapters 2, 3, and 4. For example, given the effective area of antennas, Equation 2.23 is used to derive the antenna gain

$$\text{gain} = \text{Area} \times \frac{4\pi}{\lambda^2}$$

where Area=2m^2 and λ=c/1000Mhz. Then the antenna gain is 280 or equivalently 24.5dB. Given a distance of 30miles, the propagation path loss can be determined (Equation 2.35) so that

$$(L_p)_{dB} = -36.6 - 20\log 30 - 20\log 1000 = -126 \text{ dB}.$$

Thus the antenna gain and propagation path loss figures can now be applied to the power budget equation (Equation 2.34) to derive the required transmitted power (assuming all other losses are minimal). If received power is 1 nanowatt (-90dBW), then

$$-90\text{dBW} = P_{amp} + 2(24.5\text{dB}) - 126\text{dB}$$

so that

$$P_{amp} = -13\text{dBW} \quad \text{or} \quad 50 \text{ milliwatts.}$$

It should be noted that a reduction in power from -13dBW to -90dBW represents an overall system loss of 77dB. Now concentrating on the efficiency of the transmission line, one can obtain a cable's attenuation loss factor from standard curves or tables such as Figure 5.1 extracted from Armed Services Electric Standards Agency Publication ASESA 49-2B. For a best case analysis, assume the low-loss cable RG-19A/U is used; then from Figure 5.1 it can be seen that at 1000MHz, the cable exhibits a 3.5dB loss per 100feet. Thus the total loss over the 30mile transmission distance is 5544dB. This translates to a power loss of 10^{-554}. Then applying this to Equation 3.2, the transmitter power required to yield a received signal power of 10^{-9}watts is 10^{545}watts. To analyze a microwave system one can assume the TE_{10} mode is used so that the appropriate attenuation factor can be determined. Figure 5.2 is abstracted from Blake's study on standard waveguides [Ref. 68] and presents a few representative examples of characteristics of low-loss rectangular waveguides currently used in communication

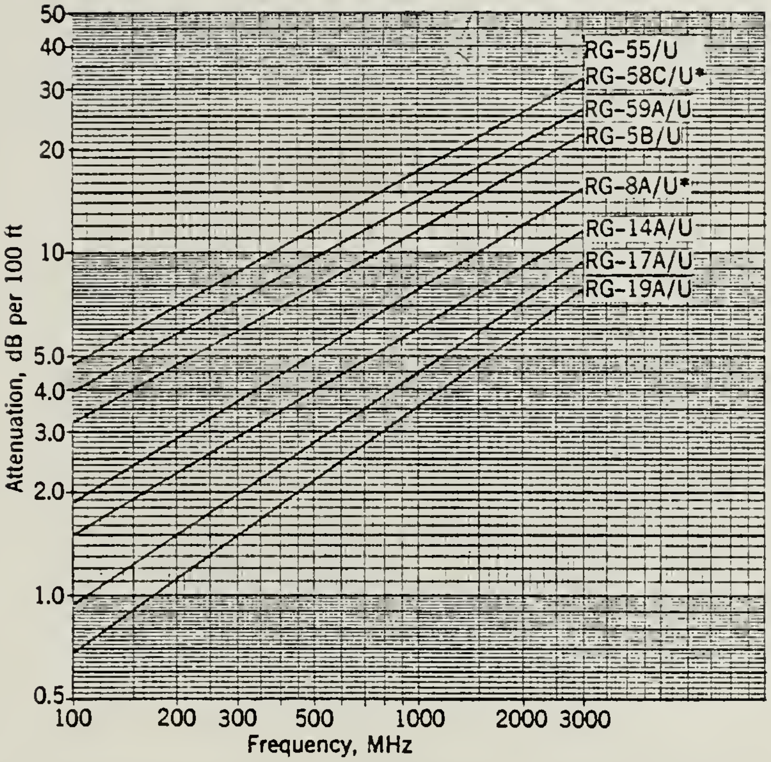
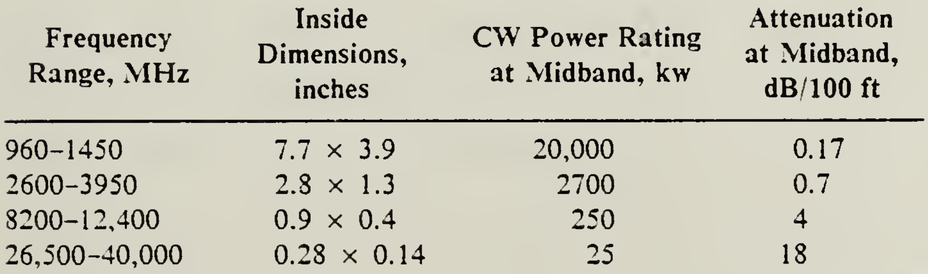


Figure 5.1 Cable Attenuation Curves.



Frequency Range, MHz	Inside Dimensions, inches	CW Power Rating at Midband, kw	Attenuation at Midband, dB/100 ft
960-1450	7.7 × 3.9	20,000	0.17
2600-3950	2.8 × 1.3	2700	0.7
8200-12,400	0.9 × 0.4	250	4
26,500-40,000	0.28 × 0.14	25	18

Figure 5.2 Waveguide Attenuation Figures.

systems. At an operating frequency of 1000MHz the attenuation factor obtained from the listing in Figure 5.2 is 0.17dB per 100feet so that over a distance of 30miles, the low-loss waveguide yields a total system loss of 270dB or 10^{-27}. The same equation used in the transmission line

analysis which relates input power to output power (Equation 3.2) is used to determine the amount of transmitter power required to produce a 1 nanowatt input signal to the receiver. Then

$$P_{out} = P_{in} L ,$$

or

$$10^{-9} \text{watts} = P_{in} \times 10^{-27}$$

so that the transmitter power requirement would be 10^{18} watts.

It should now be apparent that by drawing upon the principles of communication signal transmission, the manager of telecommunication systems can quickly conduct a fairly representative power analysis of the three kinds of communication channels. However, in addition to power constraints, other factors must be considered.

From the standpoint of selectivity and privacy, the antenna affords no general protection from intrusion, jamming, and interference (unless costly modifications are made, i.e. spread spectrum transmission) so that in a hostile electronic countermeasures (ECM) environment the antenna system could prove unreliable. The primary advantage of antenna systems is their suitability for installation onboard highly mobile platforms such as ships, aircraft, and land vehicles. Additionally, the antenna is the only link to spaceborne vehicles. A second and often equally important advantage of antenna systems is their relatively inexpensive installation costs.

It has been demonstrated in the previous chapters that the attenuation characteristics of transmission lines and waveguides exhibit an exponential relationship to distance while antenna fields are inversely related to the square of the distance. Therefore, what may hold true for one distance may not hold true for a different distance with regard to the most power efficient transmission channel. Consider the foregoing example where the transmission distance is now 5 miles. It should now be understood that for the antenna system the only change to the power budget equation will be a decrease in the propagation path loss and therefore a corresponding decrease in the required transmitted power. Since the propagation path is now 5miles (vice 30miles) the propagation path loss is

$$(L_p)_{dB} = -36.6 - 20\log5 - 20\log1000 = -110.5dB.$$

This is a reduction of 15.5dB so that the required transmitter power is reduced by 15.5dB to -28.5dBW which is 1.4milliwatts. The total waveguide loss over 5miles reduces to 45dB (0.17dB per 100 feet times 5miles). Then the transmitter power requirement is found from

$$P_{out} = P_{in}L$$

or

$$10^{-9}\text{watts} = P_{in}\times10^{-4.5}.$$

Therefore the transmitter power required for a waveguide system to produce a 1 nanowatt signal at the receiver site 5miles from the transmitter is $10^{-4.5}$ watts. Finally, the transmission line loss over the same 5mile transmission distance is found by multiplying the attenuation loss factor (3.5dB per 100 feet) by the distance (5miles) and is -920dB. Then from $P_{out}=P_{in}L$ for $P_{out}=10^{-9}$watts and L=-920dB or 10^{-92}, the input power required for a 5mile transmission line system is 10^{83}watts. To summarize, for a received signal power of 1nanowatt at the end of a 5mile transmission path, the required transmitted power for the three kinds of communication channels would be:

1. Transmission lines: 10^{83} watts (920dB loss).
2. Waveguides: 30microwatts (45dB loss).
3. Antennas: 1.4milliwatts (61.5dB loss).

Under these circumstances the waveguide is clearly the most efficient transmission channel to choose. In addition to power efficiency, the waveguide has the added benefit of signal security and greater immunity to outside interference than free space systems. However, right-of-way costs and (in some cases) inaccessible terrain limit the use of waveguides.

Reference 69 presents a similar investigation of the relationship between the transmission distance and transmitter power output requirements for a given received power level. Additionally, a comparison of the results obtained for each of the three kinds of transmission channels is graphically illustrated (see Figure 1.3). It should be noted that the actual values of the respective curves depend on the physical characteristics of each type of line, guide, or antenna at a given frequency. The dashed section of the antenna curve indicates approximate line-of-sight distances for the conventional microwave relay systems normally

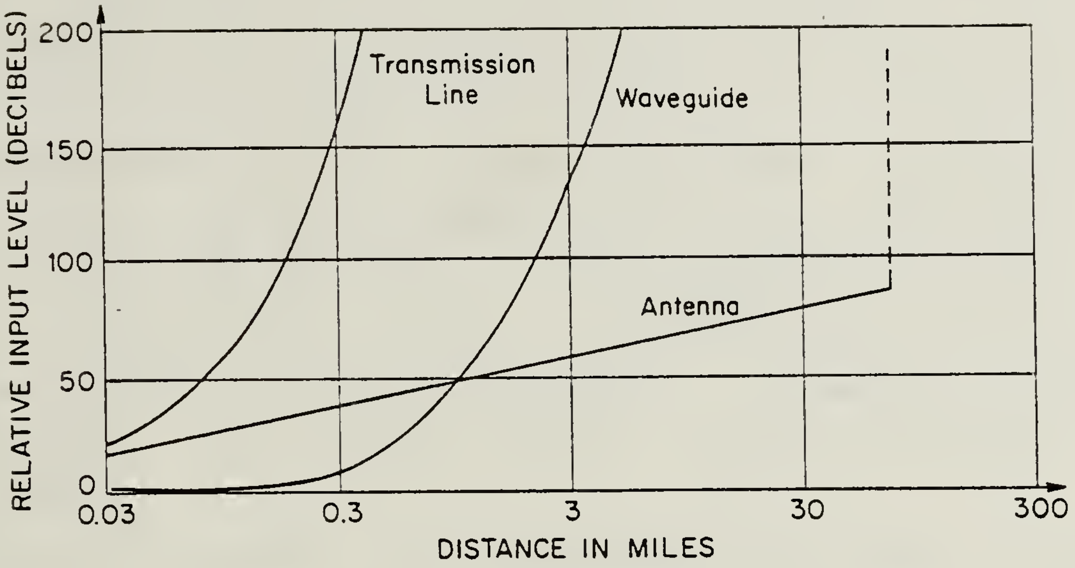


Figure 5.3 Comparative Channel Attenuation Curves.

installed. As demonstrated in the graph, at shorter distances the waveguide is the most efficient channel while the antenna performs best at the longer distances. It appears that the transmission line is generally inferior to the waveguide when comparing their respective power losses; however, the transmission line will propagate signals having frequencies as low as zero while the waveguide and the antenna have practical lower limits. For waveguides this practical lower frequency limit is near 300MHz. Theoretically, the waveguide and antenna could be designed and built to operate at the lower frequencies, but, their physical sizes would be all but impractical. For example, at a frequency of 300MHz a waveguide would be about the size of a roadway drainage culvert. The practical low-frequency limit for an antenna is on the order of 100KHz, although the international OMEGA long-range navigation system and the U.S. Navy's VLF system operate in the 10-13KHz range. In general, antennas at these frequencies are usually considered impractical. For example a dipole or half-wave antenna constructed to operate at 30KHz would be approximately 3

miles high. The transmission line, on the other hand, is extremely practical at these lower frequencies. It has the advantages of a waveguide with respect to signal privacy and immunity to outside noise and interference. Additionally, the transmission line is rugged, flexible, and inexpensive to maintain. For short distances such as a defensive position in a battlefield area, the transmission line system could be quickly set up and, unlike high gain antennas, requires no time consuming alignment. However transmission line systems are rarely used in the tactical land environment.

The major emphasis of this thesis has been to focus on the quantitative aspects of communication transmission channels (and associated subsystems) and has provided the means by which these channels can be analyzed. Too often, the communication system is viewed as a transmitter and receiver pair with little emphasis on the transmission channel. It has been demonstrated that the communication channel introduces distortion, both attenuation and time/phase shifts onto the information signal. With the rising demand for systems capable of higher information transmission rates, the communication channel can (and often does) become the limiting factor influencing overall system performance. Therefore, from a managerial perspective, it is important that considerations regarding the channel be equally weighted with considerations regarding the transmitter and receiver segments of a communication system. The purpose of this thesis has been to present the engineering fundamentals of the communication transmission channel such that the impact of managerial decisions regarding communications planning and communication systems planning become understandable.

Naval communications depend largely on free space propagation channels for information transmission. As more and more communication systems are fitted on naval ships and aircraft, the phenomena of radio frequency interference (RFI) and electromagnetic interference (EMI) become significant. A greater understanding of the system SNR change caused by an increased number of active transmitters will provide the communications manager with the tools needed to understand and perhaps solve or at least reduce the RFI and EMI effects.

Another issue which needs addressing is the Navy's heavy reliance on satellite communications. The fact that present communication satellites are vulnerable to attack by hostile forces which can negate the overhead channel has been highly publicized. Therefore, the only present alternative for ship-to-shore communications is the HF net.

Successful HF communications is highly dependent on the composition of the ionosphere where usable propagation frequencies vary seasonally as well as diurnally. Furthermore, should a high altitude nuclear detonation occur, the resulting electromagnetic pulse (EMP) would disrupt the ionosphere to such an extent that much of the radio frequency spectrum would be unusable for periods of time. The EMP would affect an area whose size would depend on factors which are beyond the control of the communication system user. The problem for the communications manager is to understand potential disruptions caused by EMP, and to select alternate strategies to be used when effects due to EMP and other interference occur.

It should be noted that no communication transmission channel is purely a transmitter-to-antenna or antenna-to-receiver connection. The two are connected by either transmission lines, waveguides, or both. The impact of environmental conditions on these elements must also be

considered, especially in an EMP environment. How does an EMP affect the SNR of a waveguide or transmission line? The optical fiber, once considered immune to EMP, is presently under examination to determine the extent of its EMP reactions.

In conclusion, the telecommunication systems manager must consider the objectives of the user. His decisions must be cost effective and produce reliable and maintainable communications. The ultimate decision must be the minimum acceptable performance level of the system, and this can be based (but not necessarily) on the system's SNR. The transmission channel is an integral part of any communication system. The SNR of the system can be significantly influenced by the transmission channel which, regardless of technology, attenuates and delays all signals. By understanding the transmission channel, which is the critical part of the communication system (especially in a jamming environment), the communications manager will more effectively devise a robust system with appropriate signal routing schemes and a system restoration plan based on priorities and protocols.

APPENDIX A

PARTIAL DIFFERENTIATION

There are many instances where a quantity w is determined by a number of other quantities. For example, the volume V of a right circular cone of radius r and height h is given by

$$V=\frac{1}{3}\pi r^2 h.$$

The values of r and h can be assigned independently of each other and once specific values have been assigned a corresponding value of V is determined. Thus V is a function of r and h. In general, if w is uniquely determined when the values of two independent variables x and y are given, it is said that w is a function of x and y and is written as:

$$w=f(x,y).$$

It should be clear that as long as the value w is not constant for all (x,y) pairs, then as the value for either x or y changes so too does the value of w. Thus the change in w is related to both x and y such that the change in w (denoted dw) is the change in the function value with respect to the change in x and the change in y. This is written as:

$$dw=\frac{\partial w}{\partial x}dx+\frac{\partial w}{\partial y}dy$$

where ∂ is the notation for the partial derivative of the function. Two of the more important rules of partial derivatives are that in general

$$\frac{\partial^2 w}{\partial x^2} \neq \frac{\partial^2 w}{\partial y^2}$$

and

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial^2 w}{\partial y \partial x}$$

where $\partial^2 w$ indicates the second derivative of the function w. [Ref. 70].

For example, if

$$w = \frac{1}{3}\pi x^2 y$$

then

$$\frac{\partial w}{\partial x} = \frac{1}{3}\pi(2x)y$$

and

$$\frac{\partial w}{\partial y} = \frac{1}{3}\pi x^2.$$

Also

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{3}\pi(2)y$$

and

$$\frac{\partial^2 w}{\partial y^2} = 0.$$

Finally note that

$$\frac{\partial^2 w}{\partial x \partial y} = \frac{\partial}{\partial x} \quad \frac{\partial w}{\partial y} \quad = \frac{1}{3} \pi (2x)$$

and

$$\frac{\partial^2 w}{\partial y \partial x} = \frac{\partial}{\partial y} \quad \frac{\partial w}{\partial x} \quad = \frac{1}{3} \pi (2x) \quad .$$

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